numgeo: A finite-element program for the simulation of hydro-mechanically coupled geotechnical processes

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Proprietary software, which is used for geotechnical simulations, is mostly not tailored to the specifics of geotechnical numerics or is not suitable for use in research due to poor extensibility. In most programs, hydro-mechanically coupled finite elements, as they are needed for the multi-phase medium soil, are implemented only in a simplified form (for example, no consideration of inertia effects). In some programs (e.g. Plaxis) so-called u-p elements are implemented, which neglect the relative acceleration between solid grains and pore fluid and may show strong oscillations in the pore water pressure at low hydraulic permeabilities. Alternative element formulations that do not exhibit these problems are not implemented in any publicly available program. To allow further development as well as the application of such element formulations, the finite element program `numgeo` *was developed by the authors. In particular, the hydro-mechanically coupled finite elements available for soil mechanics simulations will be presented, which, compared to the simplified formulation, also discretize fluid displacement. It is shown that this class of elements is advantageous for many problems since they lead to a more solvable system of equations. Some notes about special contact conditions for the fluid phase in case of contact analyses are made. Finally, the performance of the different element formulations for the simulation of shaking table tests in a centrifuge is evaluated and the influence of relative acceleration is investigated.*

1 Introduction

Most finite-element codes do not offer hydro-mechanically coupled finite-elements for the analysis of dynamic processes. Many researchers working in the field of geotechnics have therefore extended existing (closed-source) finite-element codes by so-called user-defined elements [19, 9] or have proposed work-around solutions [13, 5, 25]. In many cases these extensions work only sub-optimal due to low performance or have some shortcomings due to restrictions of extensibility of the main code.

Some finite-element codes offer simplified formulations for hydro-mechanically coupled processes such as the well-known u-p element formulation, which originally was proposed for the investigation of slow dynamic processes only. For analyses with faster dynamic action, such as the simulation of earthquakes [28] or vibratory pile driving, other element formulations are generally recommended. These shortcomings of existing finite-element codes, motivated the development of a new finite-element code named `numgeo`. `numgeo` is, however, not only focused on hydro-mechanically coupled finite-element formulations but also includes many sophisticated constitutive soil models, advanced contact discretisation and constitutive contact techniques as well as focuses on extensibility by user-defined routines. This paper gives an overview over the finite-element program itself, the advantages of the hydro-mechanically coupled element formulations available and eventually presents the numerical analysis of shaking-table tests in a centrifuge to show the performance of the code for complex boundary value problems with larger number of degree-of-freedoms (dofs).

Further applications of `numgeo`, such as the simulation of the long-term behaviour of monopile foundations for offshore wind turbines are documented in [20, 22], vibratory pile driving in water-saturated sand in [17, 21, 23] and the assessment of earthquake stability as well as air entrapment processes in [15].

2 Overview over `numgeo`

The key features of `numgeo` are displayed in Fig. 1. Having its origin in so-called user-elements developed by the authors for the commercial software `Abaqus`, `numgeo` features a vast number of hydro-mechanically coupled finite-element formulations for 2D as well as for 3D analyses. The implemented elements are suited for the simulation of

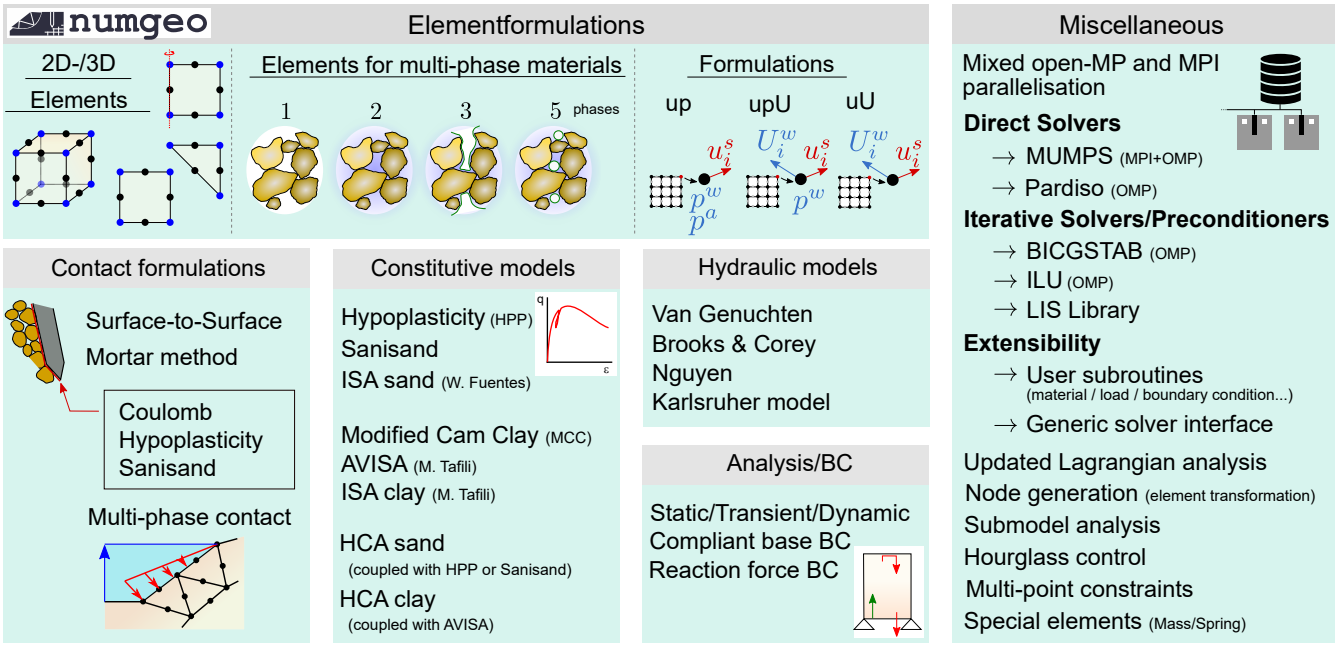

Figure 1: Overview over key features of `numgeo`

one- or two-phase flow in quasi-static, transient (time-dependent but neglecting inertia effects) as well as dynamic simulations (including the effects of inertia). The majority of these elements are based on the u-p formulation, which discretize the displacement of the solid phase and the pore fluid pressure(s) and are (for saturated soils) also implemented in some proprietary software (e.g. the program `Plaxis` or `Tochnog`). In addition, some rarely implemented elements such as the u-p-U and u-U formulations are available in `numgeo`. These elements additionally discretize the displacement of the fluid and their advantages over the u-p elements will be, amongst other, subject of this paper.

Apart from element formulations, `numgeo` includes different contact discretisation techniques for contact analyses. A surface-to-surface technique is available for 2D and 3D simulations. The mortar contact discretisation technique allows for a more accurate calculation as well as a more precise integration of the contact traction. The mortar technique is, however, only available for 2D analyses. In order to account for friction contact, Coulomb friction as well as more sophisticated friction models based on a hypoplastic model or Sanisand can be used.

Since `numgeo` is a finite element code developed for the simulation of geotechnical processes, the available constitutive models are suitable to describe the constitutive behaviour of granular or cohesive soils. Apart from "conventional" constitutive models such as the Hypoplasticity with intergranular strain or Sanisand, the high-cycle accumulation (HCA) model for sand and for clay, which is a constitutive model dedicated to the simulation of a larger number of loading cycles ($N \rightarrow 10^6$), is implemented. Novel constitutive models for clay such as the `AVISA` model (implementation by M. Tafili [26]) are available as well.

Currently concentrating on shared-memory servers, many parts of the source-code are parallelised using open-MP. For the solution of the linearised system of equations, `numgeo` includes direct [3, 4, 1] as well as iterative [18] solvers and provides a generic interface to add additional solvers.

3 Hydro-mechanically coupled element formulations

In principal, three different hydro-mechanically coupled element formulations are implemented in `numgeo`, which are the so-called u-p, u-p-U and u-U formulations. As already pointed out in Section 2, they differ in the choice of primary unknowns (discretized variables). The underlying governing equations and their discretization using the finite-element method are documented by the authors in [16, 24, 15]. Originally, the different element formulations have been proposed by Zienkiewicz [28] for fully saturated porous media. Their extension was (and still is) the subject of many studies, such as their extension to multi-phase systems with more that one pore fluid [11, 14] or double porosity porous media [8], amongst many other.

The u-p element formulation is derived assuming that the acceleration of the solid phase is identical to the acceleration of the fluid phases. Zienkiewicz and co-workers [28] have investigated for which conditions this assumption is justified and have concluded that the validity of the assumption of identical accelerations depends on the hydraulic

conductivity of the solid phase and the frequency of the propagating waves. The u-p element formulation should only be applied in case of low hydraulic conductivity and simultaneously low frequencies. For all other cases the so-called "full" formulations, such as the u-p-U and u-U formulation, are recommended.

Apart from the correct calculation of the accelerations of the different phases, the u-p-U and u-U formulation also allow for more detailed modeling of boundary conditions (and development of appropriate contact formulations) since the displacement of both skeleton and pore fluid can be described independently of each other.

Especially in 3D calculations, the computational cost for multi-phase elements increases significantly. The reason for this is on the one hand the Taylor-Hood formulation required to fulfill the LBB-condition (see e.g. [6]) in case of the u-p elements and the increased number of nodal unknowns in case of the u-U and u-p-U elements. For this purpose reduced integrated u-U elements have been developed in `numgeo`[1] (see Fig. 5). However, as known from reduced integrated single-phase elements, stabilization is required to prevent numerical instabilities (so-called zero-energy modes) associated with the rank deficiency introduced by the reduced integration. A corresponding stabilisation approach for the water phase was developed for this purpose. It is worth noting that in general fully integrated linear u-U elements should be avoided, as they are known to behave overly stiff and are prone to locking phenomena resulting in artificial pore fluid pressure oscillations. The effectiveness of this stabilization method is demonstrated in Fig. 2 and Section 5.

To demonstrate the performance of the various hydro-mechanically element formulations available in `numgeo` under well defined conditions, the wave propagation in a one-dimensional porous column is investigated. A column with a height of 10 m is loaded with a Heaviside load of 10 kPa, leading to the propagation of a compression wave. The lateral and bottom boundaries are assumed to be rigid, frictionless and impermeable. Hence, the displacements normal to the surface are blocked and the column is otherwise free to slide parallel to the lateral boundaries. At the top, the pore water pressure p^w = 0 kPa (thus, drainage at the top surface is possible) is prescribed. The same material properties as documented in [15] are used. An analytical solution for the wave-propagation has been presented by the authors in [24, 23], where a more detailed description of the considered boundary value problem (BVP) is provided as well. The convergence of the solution of the Newton's method is evaluated for both displacements as well as pore pressures. The significance of the solution control is discussed in more detail in [15]. The results in terms of excess pore water pressure at the bottom of the column and settlement of the column evaluated at its top are given in Fig. 2 as comparison with the results of the analytical approach. The naming convention of the different element formulations displayed in Fig. 2 is as follows: the number after `u` indicates the number of nodes discretizing the solid displacements, the number after `p` indicates the number of nodes discretizing the pore fluid pressure and the number after (capital) `U` indicates the number of nodes discretizing the fluid displacements. `-sat` indicates that the element assumes fully saturated conditions and `-red` that the element uses a reduced integration scheme.

As a result of the frequency of approximately 30 Hz of the propagating wave as well as the relatively high hydraulic conductivity ($k^w = 10^{-3}$ m/s), the u-p elements show an increasing divergence from the analytical results with ongoing wave propagation. The elements based on the u-p-U and u-U element formulation fit well to the analytical results for the entire time period. The relative acceleration is therefore important to be taken into account. Even the linear-interpolated u-U elements (`u4u4`, `u4u4-red`, `u8u8-3D-red`) are in good accordance with the analytical results despite the lower number of nodes and integration points (the same number of elements has been used for all simulations). The reduced integrated `u4u4-red` element performs slightly better than the fully integrated `u4u4` element which can be traced back to volumetric locking effects for the fully integrated element.

4 Contact analyses using hydro-mechanically coupled elements

Special considerations using the u-U and u-p-U element formulations in contact analyses are necessary. Both the solid and fluid displacement have to be constrained at the contact surface in order to enforce non-penetration contact conditions. Therefore, the contact traction is split into a solid contact traction and a fluid traction. This increases the complexity of the numerical procedures but allows to define different contact conditions for the individual phases. A particular advantage is the definition of a no-separation condition of contacting bodies defined solely for the fluid phase while allowing separation for the solid phase. A practical example where this is advantageous is (vibratory) pile driving in water-saturated soil, where the pile is inseparable connected with the ground water as long as no cavitation takes place. Another example is

[1] For illustration purposes consider 3D u-p elements with 27 nodes and 27 integration points per elements (see Fig. 5): the increase in computational cost is significant, even for simple models. The reduced integrated u-U elements on the other hand consist of only 8 nodes and 1 integration point and thus possess a reduced computational footprint.

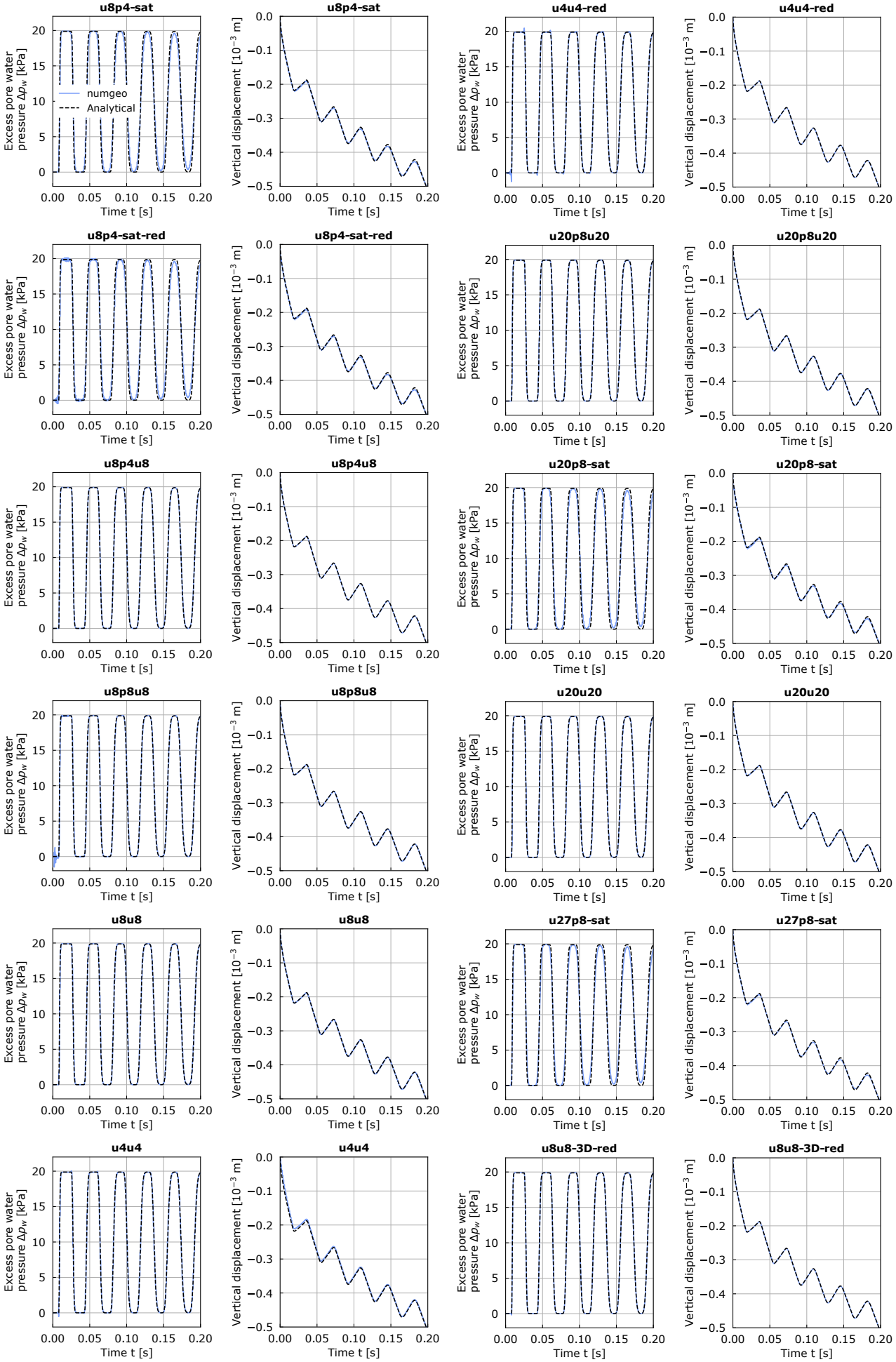

Figure 2: Excess pore water pressure at the base and displacement at the head during the one-dimensional wave propagation in a two-phase media column using an analytical approach and the various element formulations available in numgeo

the simulation of suction-buckets, where the water phase is also coupled with the bucket but the solid grains can separate from it.

Of course, a separation of the contact traction does also allow to distinguish between ”effective” and ”total” normal contact traction, which is of importance for a correct incorporation of friction contact.

Additional considerations have to be made for contact analyses using 3D serendipity elements (u20 elements). This quadratically interpolated element type is very common in finite-element codes because it is computationally more efficient than so-called bi-quadratically interpolated 3D elements which have 27 nodes (u27 elements). However, 3D serendipity elements show inferior convergence in contact analyses. Hence, numgeo uses u27 elements for 3D contact analyses, which give similar convergence ratios of the contact algorithms as linear interpolated 3D elements. Since some mesh-generators do not allow for the definition of u27 elements, numgeo allows for the automatic transformation of u20 elements to u27 elements.

5 Simulation of shaking table tests

The application of the different hydro-mechanically coupled elements in a contact analyses for a rather complex BVP is presented by means of back-calculation of shaking table centrifuge tests with a foundation upon liquefiable soil. The tests have been performed using the turner beam centrifuge at the Schofield Centre of Cambridge University and have been document in [27]. A schematic sketch of the centrifuge test in prototype scale is supplied in Fig 3. A centrifugal acceleration of 70 g has been applied. Note that the numerical analyses have been performed in the model scale and include the spin-up of the centrifuge (up to 70 g). On top of initially medium dense (relative density of 40%) Houston HN31 sand, a brass foundation was placed, which loaded the soil by 135 kPa. The sand was saturated using a fluid with a dynamic viscosity 70-times higher than that of water in order to be able to correctly capture the consolidation process of the prototype (note that otherwise, the coefficient of permeability would be 70 times higher than the prototype values). The test series includes tests under saturated[2] and unsaturated conditions. Only the saturated tests will be considered in the following.

The applied acceleration signal is shown in Fig. 4 and was applied parallel to the long side of the soil.

[2]The air in the model was removed by applying a vacuum of approximately -90 kPa. The models were then flushed with CO_2 three times to ensure complete removal of air and to reach the highest achievable degrees of saturation. The models were slowly saturated by injecting viscous pore fluid at the base of the model. The phreatic surface of the saturated models was kept at approximately 0.35 m above the ground surface.

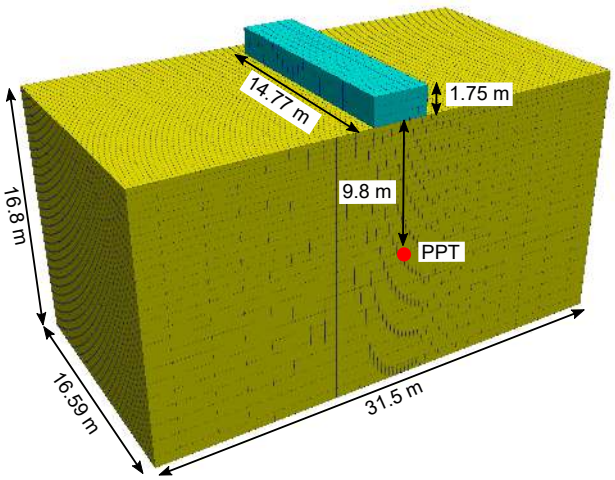

Figure 3: Finite element model and mesh used for the calculation (dimensions given in prototype scale). The location of the pore-pressure transducer (PPT) is given.

In model scale, the predominant frequency of the signal is 50 Hz. The permeability of Houston HN31 sand is approximately $1{\cdot}10^{-10}$ m^2 [12], which results in a hydraulic conductivity of 10^{-3} m/s. Hence, the relative acceleration between the solid phase and the pore fluid might be of importance in the present case.

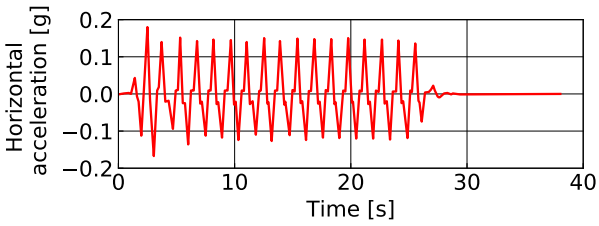

Figure 4: Acceleration-time history of the signal used in the centrifuge test (prototype scale)

Pore pressure transducers (PPT) were used to measure the pore fluid pressure during the centrifuge test. The location of the PPT used for the comparison of the measured values with the results of the numerical analyses is marked in Fig. 3.

The Sanisand constitutive model (version of 2004 [10]) has been used for the numerical analysis. The material constants for the Houston HN31 sand have been determined based on the simulation of undrained monotonic and undrained cyclic triaxial tests reported in [7] performed on Hostun-RF sand. Hostun HN31 sand is a successor and behaves mechanically similar to the Hostun-RF sand [2]. The final set of parameters is given in Table 1. A saturation level of 99% was reported for the tests [27], which translates into a bulk modulus

of the pore fluid of approximately 14 MPa.

The contact between the foundation and the soil was discretized using a surface-to-surface approach. The normal contact constraint was enforced using the penalty method with a penalty factor of 10^5 kN/m^3. In case of the simulations using the u-U or u-p-U element formulations, the penalty factor was set to 10^5 kN/m^3 for both phases. A previous sensitivity study showed that the solution does not depend on the choice of the penalty factor. Friction was considered using a simple Coulomb model with a friction coefficient of $2/3\varphi_c$, with $\varphi_c = 32°$. The tangential stiffness of the Coulomb model was found to influence the results of the simulations strongly since the foundation moves significantly in lateral direction during the shaking. A value of 5000 kN/m^2 was set eventually.

An updated Lagrangian framework has been used for all analyses. For the reduced integrated elements a sensitivity study regarding the Hourglass stiffness was performed. Values of 200 kPa and 500 kPa lead to no change in the results of the simulations compared to 100 kPa, which was used eventually. The different element formulations used in this study are given in Fig. 5.

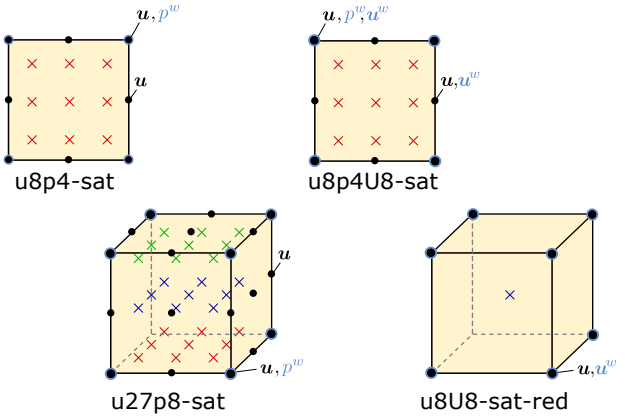

Figure 5: Different element formulations used for the simulation of the shaking table tests in Section 5. p^w is the pore fluid pressure, $\boldsymbol{u}$ and $\boldsymbol{u}^w$ denote the displacements of the solid phase and the pore fluid phase, respectively.

In order to demonstrate the capabilities of the developed finite element code, the simulations have been performed using different element formulations. To evaluate the influence of the relative acceleration between the solid and the fluid phase, 2D simulations using the u-p (`u8p4-sat` element, see Fig. 5) and the u-p-U (`u8p4u8-sat` element, see Fig. 5) formulation have been performed. The settlement of the foundation during the shaking is given in prototype scale in Fig. 6 for the two simulations and the values measured in the experiment. Both simulations predict the settlement during the shaking very well. After the shaking

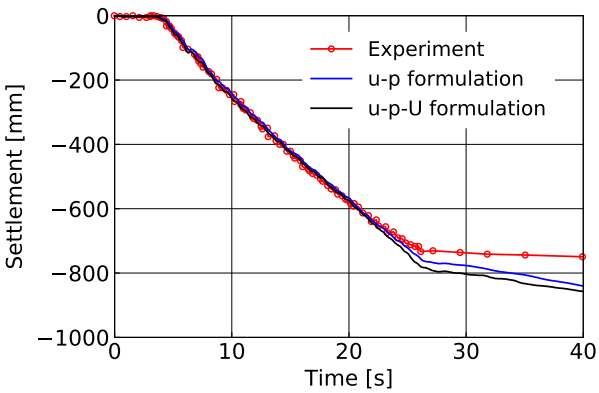

Figure 6: Settlement of the foundation during the shaking for the values measured in the experiment and 2D simulations using the u-p and and the u-p-U element formulations

stops at approximately 25 s, both simulations show a continuation of settlement which is absent in the experiment. This settlement occurs due to the consolidation process following the shaking. Both element formulations perform similar but the u-p element formulation shows slightly less settlement compared to the u-p-U element formulation. Hence, the relative acceleration is influencing the settlement of the foundation only slightly.

The comparison of pore fluid pressure ratio[3] r_u measured at the location of the PPT marked in Fig. 3 is given for both simulations in Fig. 7. Compared to the experiment, the simulation results capture the initial increase in excess pore water pressure well but overestimate the subsequent dissipation. There is no noticeable difference between the simulations themselves. Hence, the excess pore fluid pressure is not strongly influenced by the assumption of zero relative acceleration (u-p formulation). It can be concluded that despite the high frequency of the earthquake signal and the high hydraulic conductivity of the soil the influence of relative acceleration is small. This is in accordance with previous investigations of the authors using an analytical approach to quantify the influence of relative acceleration reported in [24, 23].

Further simulations were carried out to verify the assumption of simplification to a 2D system and the performance (and robustness) of the reduced integrated u-U elements. As can be seen from Fig. 3, the foundation does not have the same length as the soil volume. Thus, a 2D analysis might not be applicable. Additional analyses using different 3D element formulations based on the u-p and u-U formulation have been performed. The calculated settlements are compared to the measured ones in Fig. 8. Both simulations are in good agreement

[3] The (excess) pore fluid pressure ratio $r_u = \Delta p^w / \sigma'_{v0}$ relates the excess pore fluid pressure Δp^w to the initial vertical effective stress σ'_{v0} without the foundation-induced stresses.

Table 1: Material parameters of Sanisand for Hostun-RF/Houston HN31 sand

e_0	λ	ξ	M_c	M_e	m	G_0	ν	h_0	c_h	n_b	A_0	n_d	z_{max}	c_z
[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]	[-]
1.103	0.122	0.205	1.375	0.942	0.05	20	0.05	10.5	0.55	1.0	0.9	1.9	10.0	1000

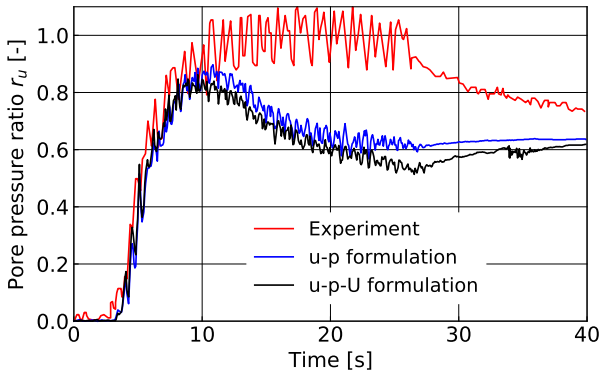

Figure 7: Pore pressure ratio r_u measured at the location of the PPT marked in Fig. 3 during the shaking for the values measured in the experiment and 2D simulations using the u-p and and the u-p-U element formulations

with the measurements. Compared to the 2D simulations, a better agreement with the experiments for the final settlements and the post-seismic consolidation phase is achieved. The 3D simulations predict less displacement of the foundation compared to the 2D simulations which is traced back to the correct reproduction of the foundation length, which is ∼1.8 m shorter than the soil body.

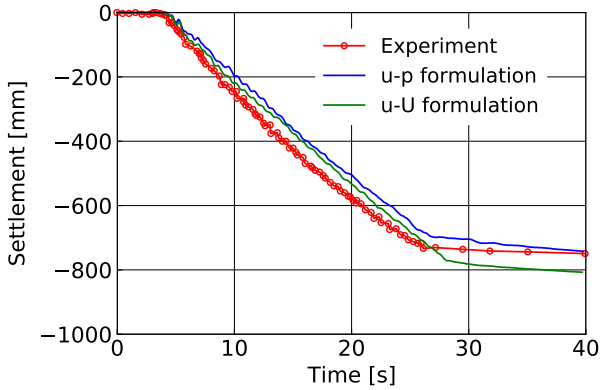

Figure 8: Settlement of the foundation during the shaking for the values measured in the experiment and 3D simulations using the u-p and and the u-U element formulations

A comparison of the deformed configuration after the excitation ($t = 40$ s) for the simulation with the reduced integrated u-U elements and the experiment is depicted in Fig. 9. As in the experiment, the foundation shows larger settlement than the adjacent ground due to partial liquefaction below the foundation. The ground adjacent to the foundation settles more than the free field, which is observed for the simulation as well as for the experiment.

From a computational cost point of view, using the reduced integrated u-U elements ($0.36 \cdot 10^6$ degrees of freedom (dofs) in total and 1 integration point per element), the calculation time was reduced from ∼ 10 days (for the u-p formulation with about $0.45 \cdot 10^6$ dofs but 27 integration points per element) to ∼ 2 days (the time required for each iteration reduced from ∼ 970 s to ∼ 125 s)[4].

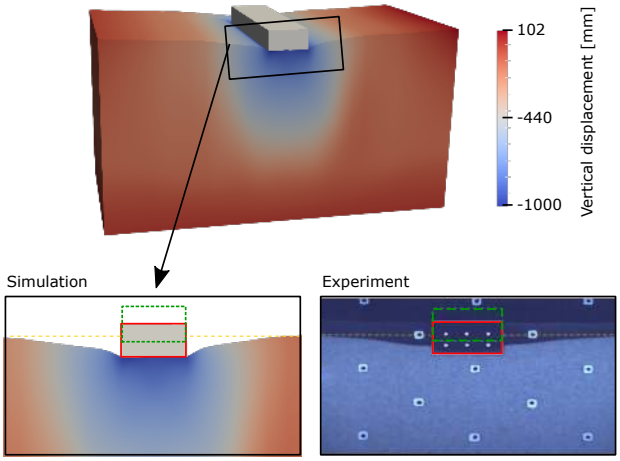

Figure 9: Deformed configuration after the excitation ($t = 40$ s) for the simulation with the reduced integrated u-U elements and the experiment (picture from [27])

6 Conclusions

The finite-element code `numgeo` offers various hydro-mechanically coupled finite-element formulations allowing for the analysis of saturated and unsaturated porous media under dynamic action. The influence of the relative acceleration between the solid and the fluid phase, neglected by the commonly applied u-p element formulation, can be incorporated using the u-p-U and u-U elements available in `numgeo`.

A one-point integrated and hourglass-enhanced u-U element was introduced, which is superior to elements based on the u-p and u-p-U element for-

[4] Using 6 cpus on a Intel(R) Xeon(R) CPU E5-2690 v2 3.0 GHz

mulation in terms of numerical efficiency. In addition, special contact constraints for the fluid displacement in case of contact analyses were implemented. The performance of the different element formulations was evaluated for a benchmark problem and the analysis of shaking table tests in a centrifuge. Both the development of settlements measured at one point as well as the deformation pattern was well captured by the simulations. The magnitude of excess pore fluid pressure (measured at one location of the model) was underestimated by approx. 20 % in all simulations and a too fast dissipation of excess pore fluid pressure was predicted.

numgeo is scheduled for a public release in the fall of 2021 (as freeware). A release of numgeo as open-source software is planned for 2022.

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