BACK-CALCULATIONS OF CENTRIFUGE TESTS ON PILE GROUPS SUBJECTED TO HIGH-CYCLIC LOADING

Jan Machaček, Author, Institute of Geotechnics, Department of Civil and Environmental Engineering, Technical University of Darmstadt, Darmstadt, Germany, jan.machacek@tu-darmstadt.de

Frederik Brosz, Contributor, Institute of Geotechnics, Department of Civil and Environmental Engineering, Technical University of Darmstadt, Darmstadt, Germany, frederik.brosz@tu-darmstadt.de

Patrick Staubach, Contributor, Chair of Soil Mechanics, Foundation Engineering and Environmental Geotechnics, Ruhr-Universität Bochum, Bochum, Germany, patrick.staubach@rub.de

Torsten Wichtmann, Contributor, Chair of Soil Mechanics, Foundation Engineering and Environmental Geotechnics, Ruhr-Universität Bochum, Bochum, Germany, torsten.wichtmann@rub.de

Hauke Zachert, Contributor, Institute of Geotechnics, Department of Civil and Environmental Engineering, Technical University of Darmstadt, Darmstadt, Germany, hauke.zachert@tu-darmstadt.de

ABSTRACT

The majority of heavy or strongly loaded structures, for instance high-rise buildings, large span bridges or gas storage tanks are founded on pile groups. However, current design codes and guidelines do not provide a standard procedure for their design under lateral dynamic and high-cyclic loading arising from e.g. wind, temperature or earthquake loads. To establish appropriate design strategies, a better understanding of the pile-soil, pile-pile and pile-cap interaction is required. This paper is focused on finite element back-calculations of centrifuge tests on single piles and pile groups subjected to up to 500 lateral load cycles. For the finite element simulations, a special calculation procedure is applied, combining two kinds of constitutive models, a conventional (Hypoplasticity with Intergranular Strain extension) and a special high-cycle accumulation (HCA) model. The parameters of the constitutive models are calibrated based on static and cyclic laboratory tests on the sand used in the centrifuge tests. The performance of the applied numerical procedure is evaluated based on a detailed comparison of the simulation results with the measured data from the centrifuge tests. Overall the applicability of the used numerical strategy for simulating the behavior of pile groups subjected to high-cyclic loading is shown.

Keywords: pile groups, centrifuge tests, high-cyclic loading, back-analysis, high cyclic accumulation (HCA) model

1. INTRODUCTION

Despite extensive research, the prediction of a pile group's response due to dynamic loading remains a challenging task. A current field of research in this context is the investigation of numerical strategies for the simulation of the pile group's behavior under lateral (high-)cyclic loading. For this purpose, a validation based on field measurements on pile groups would be particularly desirable. However, full-scale tests and their instrumentation are very complex and expensive. In addition, the soil conditions *in-situ* are often not known in such detail, that they satisfy the high demand of accuracy needed for a validation of the soil models and the numerical simulation. Solutions for this are 1-g model tests and N-g centrifuge tests. One of the big advantages of centrifuge tests is the replication of the prototype initial stress conditions in the soil. Hence, highly instrumented pile groups can be tested in model scale under realistic initial stress conditions.

This paper is structured as follows: first, the centrifuge setup and test program are discussed in section 2. The concept of the used numerical calculation procedure is covered in section 3, followed by the material parameter calibration of the used sand in section 4. The finite element models applied in the back-calculations of the centrifuge tests are presented in section 5. Finally, the results of the simulations are compared with the results from the experiments in section 6.

2. CENTRIFUGE TESTS

In order to study the effects of cyclic loading, centrifuge tests on single piles and pile groups embedded in sand were carried out at the University of Western Australia (Niemann et al. (2018)). A detailed description of the model tests and their results are provided in Niemann (2020). The test setup is briefly summarized in the following.

2.1 Test setup

The pile group consists of five circular hollow piles and a cruciform shaped cap, see Figs. 1, 2a) and 2c). The acceleration of the centrifuge was set to 100 g. Hence, for prototype scale, the dimensions in Fig. 1 give: outer pile diameter = 1.0 m, inner pile diameter = 0.8 m, pile length = 13 m and thickness of the pile cap = 1.0 m. The aluminum piles were rigidly connected to the pile cap (made of stainless steel) by threaded connections with the possibility to vary pile spacing and thus test different pile arrangements. The surface area of the piles was sandblasted to achieve a skin friction comparable to the one of bored piles. The pile tips were welded to avoid sand entering the piles during the installation process.

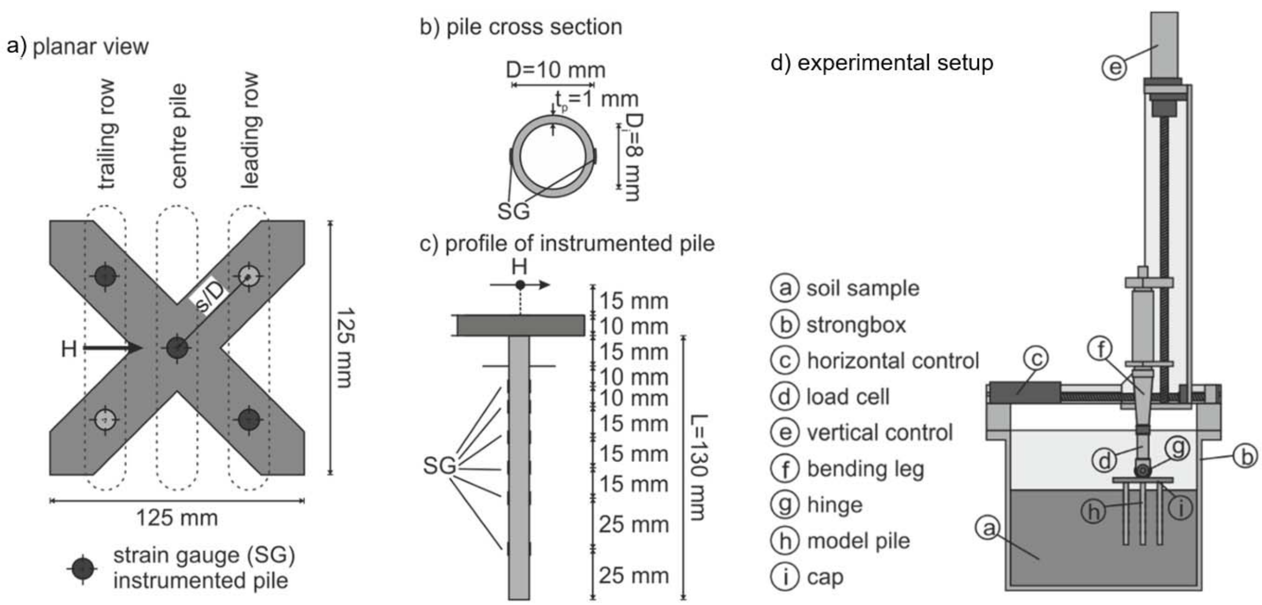

Fig. 1. Centrifuge tests: Pile group geometry in model scale, instrumentation and experimental setup (after Niemann 2020).

The horizontal load was applied at a height of 15 mm (in model scale) above the pile cap by a two-dimensional electrical actuator by a bending leg (Fig. 1d)). The load connection point was hinged. Stabilizing vertical loading was omitted in the tests in order to reach maximum horizontal displacements. They were measured by an optical encoder in the horizontal axis of the actuator.

2.2 Test sand and sample preparation

Fine-to-medium-grained sub-angular silica sand was used in the centrifuge tests with its index parameters: median grain size d_{50} = 0.19 mm $^{A)}$, coefficient of uniformity C_U = 1.9 $^{A)}$, grain density ρ_s = 2.65 g/cm$^{3\ A)}$, minimum void ratio e_{min} = 0.525 $^{B)}$, maximum void ratio e_{max} = 0.787 $^{B)}$ and critical state friction angle φ_{cv} = 33° $^{B)}$ (A: after Bienen *et al.* 2012, B: determined in laboratory tests at RUB 2021).

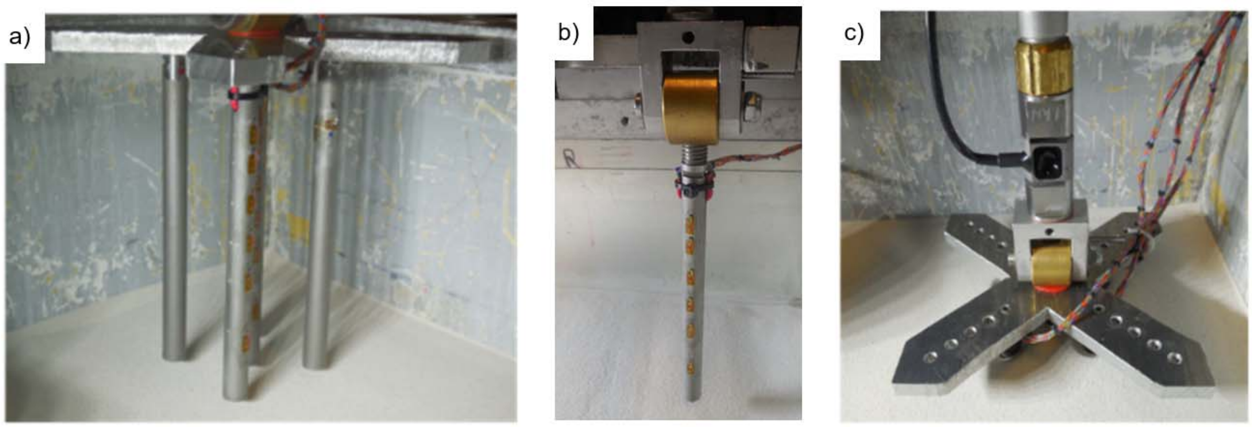

Fig. 2. Experimental setup of the pile tests: a) pile group before installation, b) single pile before installation, c) installed pile group with hinge and load cell (after Niemann 2020).

The air pluviation technique was used to prepare a medium dense sample in the model container. Two cone penetration tests (CPTs) showed similar penetration resistance and thus, a homogenous sample, see Niemann (2020). The calculated relative density derived from mass and volume of the sample was 0.45, which is in accordance with the averaged relative density estimated from the CPTs (0.47 and 0.49).

2.3 Pile installation and test program

The pile installation was realized by pushing the piles into the soil at 1 g (before the spin-up of the centrifuge). According to Li *et al.* (2010) such an installation reflects *in-situ* conditions of a bored pile. Both single piles and pile groups were tested in one-way and two-way cyclic loading tests reaching up to 500 load cycles at 100 g. To provide a reference for the determination of the cyclic load magnitude, a monotonic single pile test was performed which resulted in an ultimate lateral force measured at the pile head of H_{ult} = 2 MN (see section 6.1). The cyclic load magnitude ζ_b is defined as:

$$\zeta_b = \frac{\mathrm{H}_{max}}{\mathrm{n}_p\ \mathrm{H}_{ult}}$$

where H_{max} is the maximum applied horizontal load, n_p denotes the amount of piles in the pile group and H_{ult} is the ultimate lateral load measured at the pile head in the monotonic single pile test.

For the present study, both a single pile and a pile group subjected to one-way cyclic loading with a load magnitude ζ_b = 0.2 are considered, resulting in H_{max} = 400 kN for the single pile and H_{max} = 2000 kN for the pile group. The pile axis distance of the considered pile group is S = 3 m (S/D = 3).

3. GENERAL CONCEPT OF THE HCA MODEL

The application of conventional numerical soil models for the simulation of (high-)cyclic loading is limited to a relative low number of load cycles (N < 100). Reasons for this are their inability to capture the cumulative effects during high-cyclic loading and the accumulation of numerical errors with increasing number of cycles as well as the large calculation effort. Niemunis *et al.* (2005) developed a high-cyclic accumulation (HCA) model for the simulation of long-term soil deformation due to high-cyclic loading, which was calibrated on many laboratory tests by Wichtmann *et al.* (2005). In Zachert *et al.* (2020), Machaček *et al.* (2018) and Staubach *et al.* (2021) the HCA model was validated by the back-analysis of field tests and model tests which showed good agreement between measurements and FE simulations. So far, the HCA model has not been used for the recalculation of pile groups and for boundary value problems with comparatively small number of cycles (N < 1000). Recently, the HCA model was extended by an adaptive strain amplitude by Staubach *et al.* (2021a), accounting for the changes in strain amplitude during the high cyclic loading.

The general calculation procedure of the HCA model comprises a low-cycle and a high-cycle mode (Fig. 3). During the first few load cycles (low-cycle mode), a conventional soil model e.g. a hypoplastic model is used to calculate the soil response. Here, a hypoplastic model of von Wolffersdorff (1996) with Intergranular Strain extension (Niemunis and Herle 1997) is used in the back-calculations of the centrifuge tests. The strain amplitude ε^{ampl} of the HCA model is calculated from the third load cycle since the deformations calculated in the first two cycles differ significantly from the following cycles. During the high cycle mode, the HCA model predicts the accumulation of soil deformations based on a given cycle package while the external load F is kept constant on the average value F^{av}. A detailed description of the HCA model can be found in Niemunis *et al.* (2005).

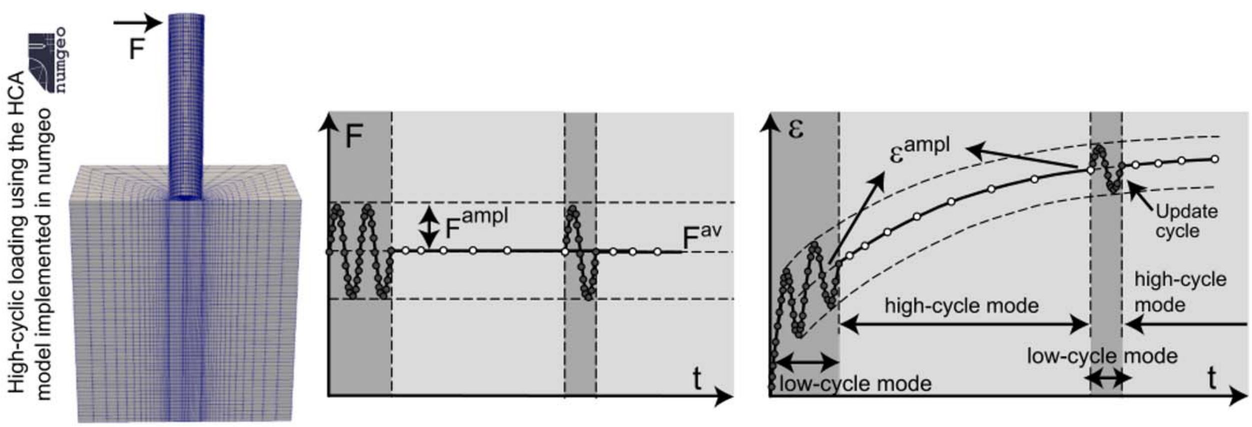

Fig. 3. FE simulation of high-cyclic loading using a combination of low-cycle and high-cycle calculation phases (after Staubach *et al.* 2021).

4. MATERIAL PARAMETER CALIBRATION

For the hypoplastic model with Intergranular Strain extension, an automatic calibration (AC) tool developed by the first author is used. The AC program aims to simplify and speed up the calibration process and, in particular, aims to reduce the application hurdles when using advanced constitutive soil models. The automatic calibration program combines the finite-element program `numgeo` (Machaček and Staubach 2021, www.numgeo.de) with heuristic optimization strategies such as Differential Evolution and Particle Swarms Optimization. It allows an easy import of all test data and the subsequent AC with the identical implementation of the constitutive model which is also used for the simulation of boundary value problems.

The parameters of the hypoplastic model are calibrated based on three oedometric compression tests[1] and five drained monotonic triaxial tests[2]. The Intergranular Strain parameters of the Silica Sand are calibrated by back calculation of one undrained cyclic triaxial test. The parameters of the hypoplastic model with Intergranular Strain extension are summarized in Table 1. The results of the calibration are provided by means of comparison of experimental and numerical data for the oedometric compression tests and the drained monotonic triaxial tests in Fig. 4 and for the cyclic triaxial test in Fig. 5.

Table 1. Parameters of the hypoplastic model (top row) and the Intergranular Strain extension (bottom row) for the Silica Sand used in the centrifuge tests.

φ_c [rad]	h_s [GPa]	n	e_{i0}	e_{c0}	e_{d0}	α	β
0.516	54.86	0.2436	0.932	0.81	0.442	0.2584	0.2256

m_T	m_R	R	β_R	χ
1.6	3.2	$2 \cdot 10^{-4}$	0.03	1.5

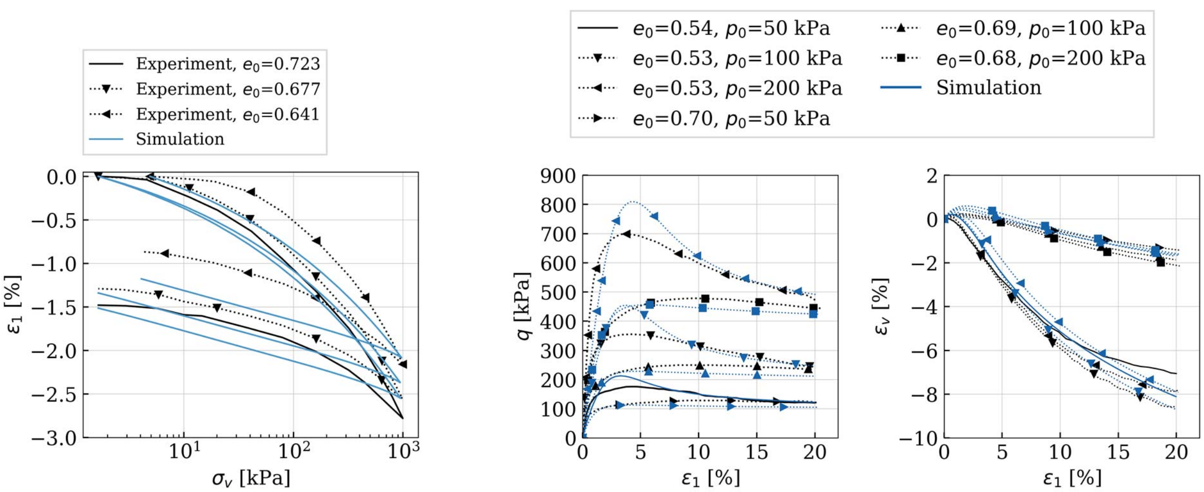

Fig. 4. Comparison of simulations (blue lines) and test results (black lines) of oedometric compression tests (left) and drained monotonic triaxial tests (right) on samples with different initial densities and different initial mean effective stresses.

The material constants of the HCA model are calibrated based on three cyclic drained triaxial tests performed on Silica Sand. For the calibration of all parameters of the HCA model based on laboratory tests, at least eleven tests would be required. In the present case, the parameters for the functions considering the influence of the void ratio (parameter C_e), the stress ratio (parameter C_Y) and the mean effective stress (parameter C_P) are estimated based on the median grain size, the uniformity coefficient and the minimum void ratio following the procedures described in Wichtmann *et al.* (2015). The parameter C_{ampl}, determining the influence of the strain amplitude on the strain accumulation, is calibrated using three tests with varying stress amplitude (but constant stress ratio). The results of these tests are given in Fig. 5 in terms of accumulated strain divided by the functions f_{ampl}, f_e, f_Y and f_p versus number of load cycles N. The accumulated strain is divided by the functions in order to detach the influence of the strain amplitude, the

[1] The oedometric compression tests were performed at the Chair of Soil Mechanics, Foundation Engineering and Environmental Geotechnics, Ruhr-Universität Bochum.
[2] The drained monotonic triaxial tests were performed at the Institute of Geotechnical Engineering and Construction Management of the Hamburg University of Technology and published in Chow et al. (2019).

void ratio, the stress ratio and the mean effective stress from the results. The plots in Fig. 5 are used for the determination of the parameters of cyclic history C_{N1}, C_{N2} and C_{N3} of the HCA model. The final set of parameters of the HCA model is given in Table 2.

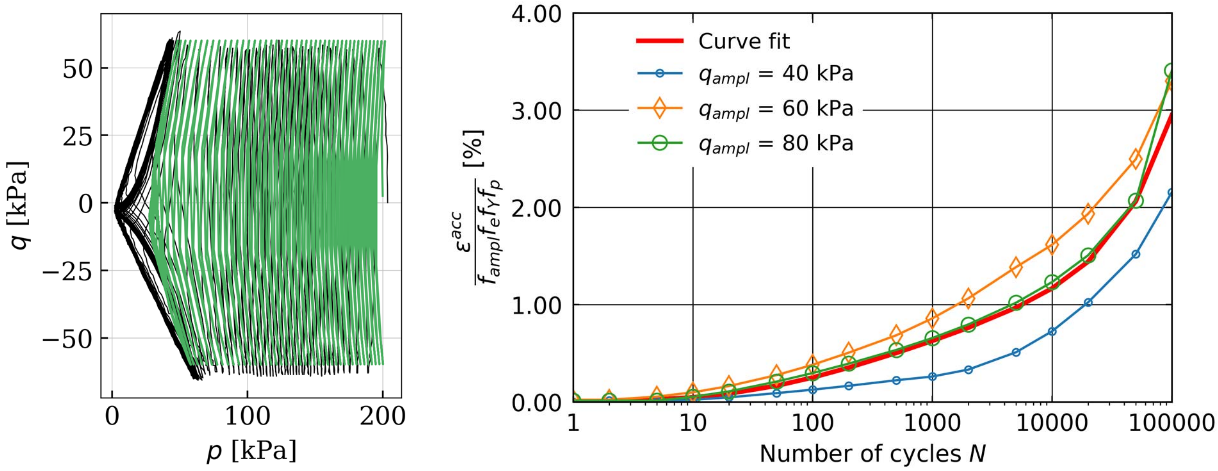

Fig. 5. Left: Comparison of simulation (green) and test result (black) of one undrained cyclic triaxial test. Right: Strain accumulation divided by the functions f_{ampl}, f_e, f_Y and f_p of the HCA model versus number of applied load cycles for three drained high-cyclic triaxial tests with different stress amplitude. The plots are curve-fitted in order to obtain the parameters C_{N1}, C_{N2} and C_{N3}.

Table 2. Parameters of the HCA model used for the Silica Sand.

C_{ampl}	C_e	C_p	C_Y	C_{N1}	C_{N2}	C_{N3}
1.74	0.42	0.469	1.63	0.000875	0.0305	0.0000619

5. FINITE ELEMENT MODEL

The geometry of the centrifuge tests (section 2) is modelled in prototype scale. The FE models of the single pile and the pile group are displayed in Fig. 6 and Fig. 7, respectively. Due to the unidirectional lateral loading, it is sufficient to model only one half of the model test exploiting the symmetry of the boundary value problem. Reduced integrated, hourglass-enhanced eight-noded brick elements are used for the discretization. The total number of elements is 115,300 for the FE model of the pile group. The dimensions of the modelled soil body are 24.5 m in longitudinal direction and 12.25 m in transverse direction. The height of the modelled soil body is 17.5 m. The mesh is refined at critical parts of the model (e.g. area around the piles). The load is applied 1.5 m above the top of the pile cap (Fig. 7) which corresponds to the axis of the load application point in the centrifuge tests (cf. Fig. 1). Due to the large deformations reported in the tests, an updated Lagrangian (geometrically non-linear) formulation is employed. For this purpose, the Jaumann-Zaremba stress rate is used. An enhanced HCA model with adaptive strain amplitude was employed. Contrary to the conventional HCA model, the strain amplitude is not constant during the high-cycle mode but updated at given times. In the present study an update of the strain amplitude was performed after 10, 50, 100 and 200 cycles.

The contact between the piles and the soil is discretized using a surface-to-surface method. The contact constraints are enforced by the penalty method. The penalty factor in normal direction is twenty times the trace of the material stiffness of the continuum element closest to the contact point. A stabilizing compressive pressure of 1 kPa was applied on the ground surface in order to avoid zero pressure in the soil close to the surface. This additional surface pressure does not influence the results of the simulations but improves the convergence of the solution, which was proven by simulations with smaller surface pressures.

Based on the CPT-results reported in Niemann (2020), a depth-dependent distribution of the initial void ratio $e_0(z)$ is used:

$$e_0(z) = e_{max} - (-0.445 \cdot exp[-1.105 \cdot z] + 0.468)(e_{max} - e_{min})$$

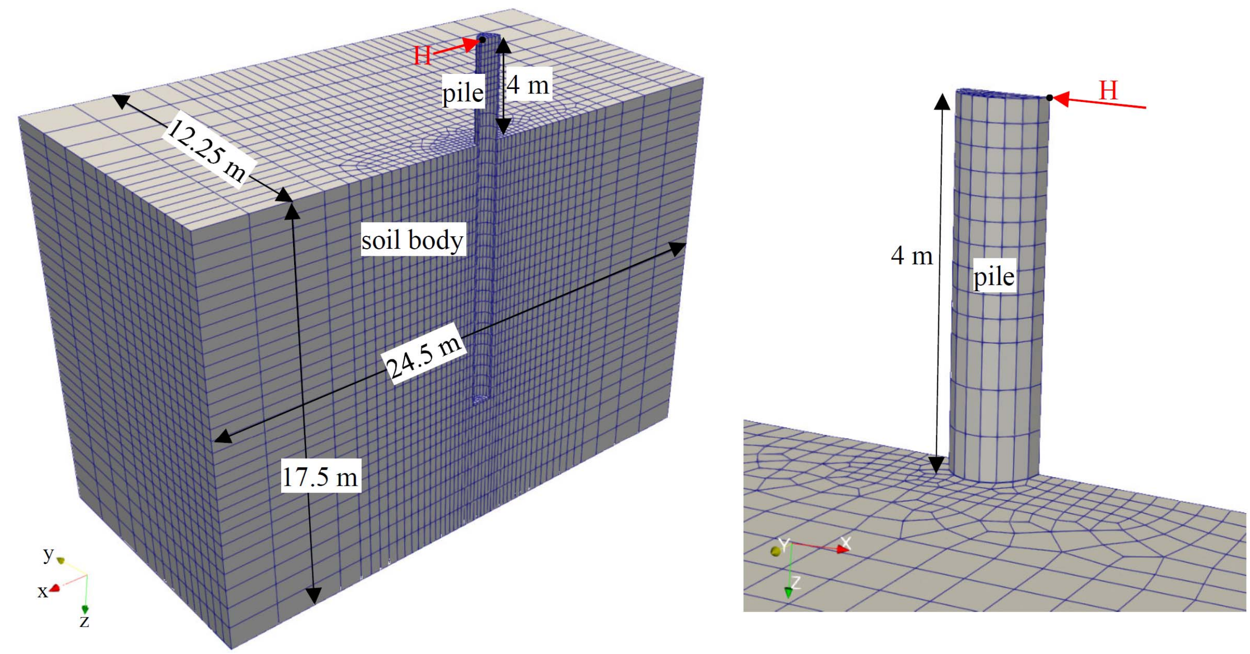

Fig. 6. Finite element model of the single pile (exploiting symmetry).

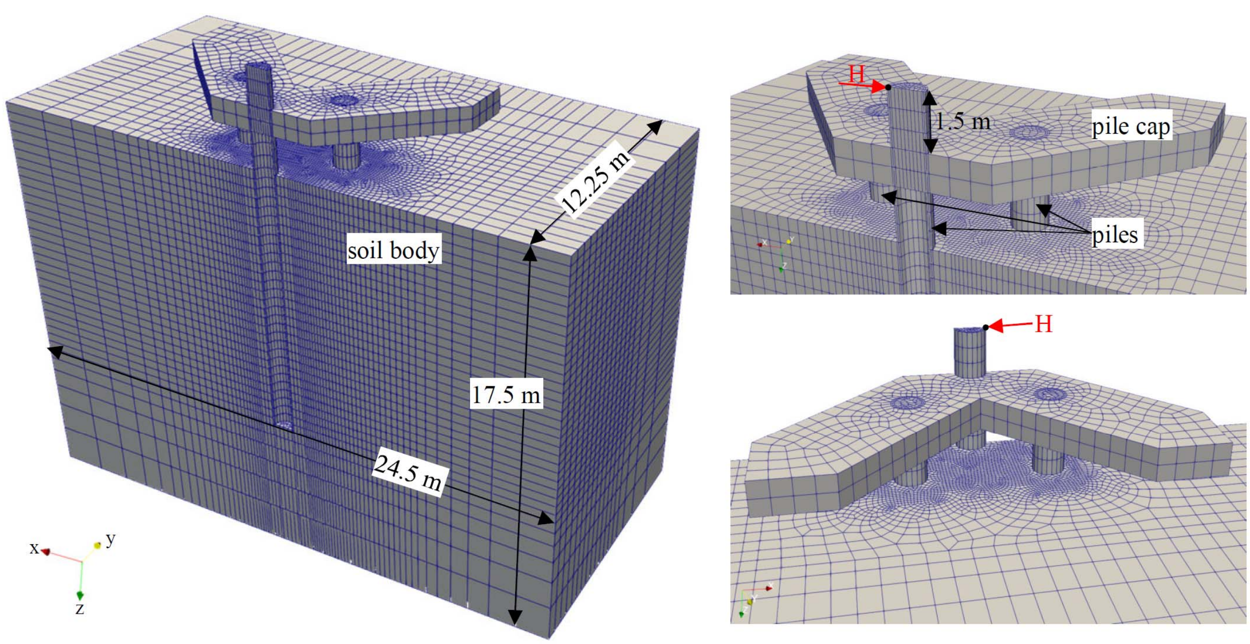

Fig. 7. Finite element model of the pile group (exploiting symmetry).

6. SIMULATIONS

The back-analyses of the centrifuge tests are performed using the finite element code `numgeo`[3]. The monotonic single pile test and the corresponding simulation are discussed in section 6.1. Experiments and simulations of the cyclically loaded single pile and the pile group are discussed in section 6.2 and in section 6.3, respectively. Results of the measured horizontal pile head displacements due to cyclic loading are presented in Fig. 9 for the single pile and in Fig. 10 for the pile group by means of colored red circles. These red circles represent the pile head displacement measured at the maximum and minimum stress level of each cycle. The displacement curves (cf. right hand side in Figs. 9 and 10) are derived from the load-displacement curves (cf. left hand side in Figs. 9 and 10). The deformations from the cyclic loading are given relative to the deformation of the first irregular cycle measured at average load H_{av}.

6.1 Results of the monotonic single pile test and simulation

The displacement-controlled experiment is reaching a maximum horizontal force of approximately 2200 kN at a very large horizontal pile head displacement of about 1.15 m which is even exceeds the pile diameter of 1.0 m (Fig. 8). The load-displacement behavior is highly non-linear. In the simulation a pile head displacement of approx. 0.32 m (approx. 30 % of the pile diameter) is reached, which corresponds to a load of 1320 kN. From this point on, the simulation terminates due to convergence problems. For a further calculation, a numerical method for large deformations would be required. The simulation and the experiment are in very good agreement within the range of the back-analysis (Fig. 8). As in Niemann (2020), the ultimate lateral force of the monotonic single pile test was set to H_{ult} = 2 MN for the calculation of the load magnitude ζ_b (section 2.3).

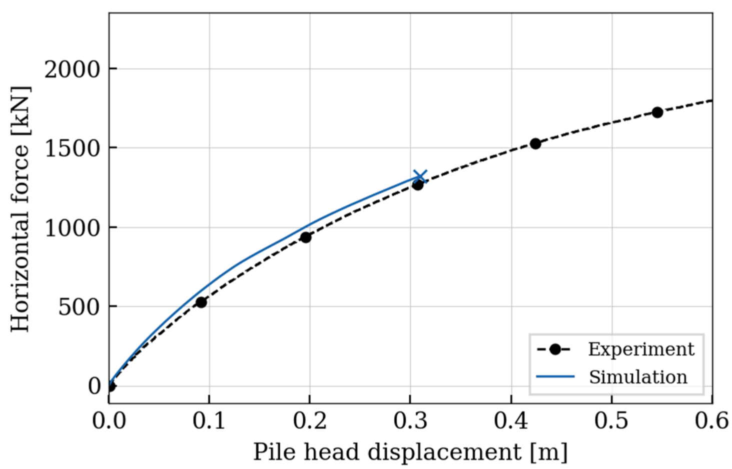

Fig. 8. Comparison of horizontal force at pile head vs. pile head displacement of the monotonic single pile test for experiment (dotted line, black) and simulation (solid line, blue).

6.2 Results of the cyclic single pile test and simulation

The results of the FE simulation of the cyclic single pile test and the experimental results are shown in Fig. 9. Both the experiment and the simulation show an increase of the accumulated pile head displacements with increasing number of cycles. The maximum calculated horizontal pile head displacement after the first load cycle is 10.5 cm and is in good agreement with the pile head displacement measured in the experiment (cf. left-hand side in Fig. 9). During the cyclic loading, a linear trend of the accumulated pile head displacement is observed for the experiment, which is matched well by the simulation within the range of < 100 load cycles. With further increasing number of cycles, the deviation of both curves increases (Fig. 9, right hand side). The load-displacement behavior at the pile head is reproduced well by the simulation,

[3] `numgeo` (see Machaček (2020), Machaček & Staubach (2021), Staubach et al. (2021) and www.numgeo.de) is an in-house finite-element program developed for the simulation of geotechnical boundary value problems.

resulting in a satisfactory prediction of the horizontal pile head stiffness (Fig. 9, left hand side). Considering the applied simplified calibration procedure of the HCA model, the simulation predicts the accumulated pile head displacements and the horizontal pile head stiffness behavior of the single pile reasonably good.

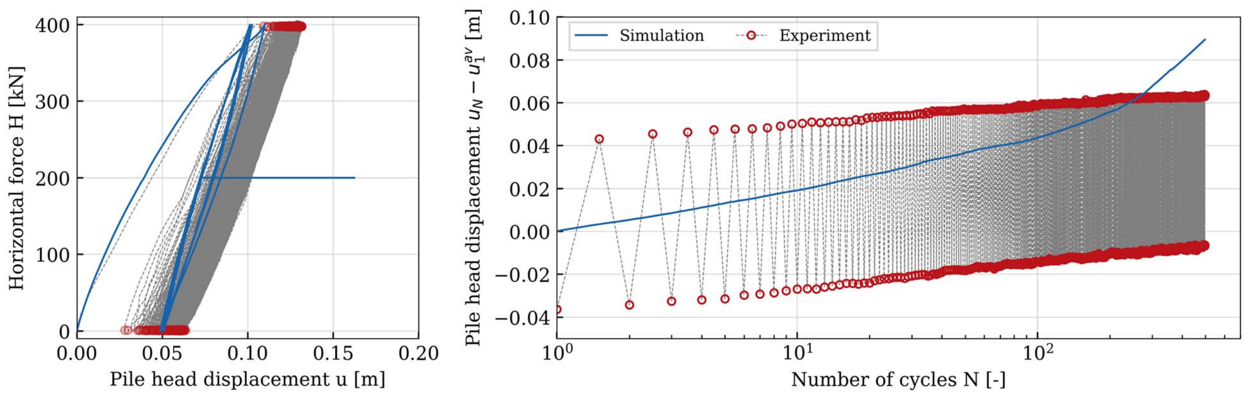

Fig. 9. Comparison of experiment and simulation for the cyclically loaded single pile: horizontal force at pile head vs. pile head displacement (left); Pile head displacement vs. number of load cycles (right). The red circles represent the pile head displacements measured at maximum and minimum load level of each cycle. The deformations from the cyclic loading are given relative to the deformation of the first irregular cycle measured at average load H_{av}.

6.3 Results of the cyclic pile group test and simulation

In contrast to the single pile, the simulation of the load-deformation behavior evaluated at the load application point of the pile group is clearly stiffer compared to the experiment (Fig. 10, left hand side). After the first load cycle, the simulation predicts a displacement of approx. 8 cm at the load application point, while the measurement of the experiment yields 17 cm (Fig. 10, left hand side). Despite this discrepancy, the simulation of the accumulated displacements with increasing number of cycles is in good agreement with the measurements.

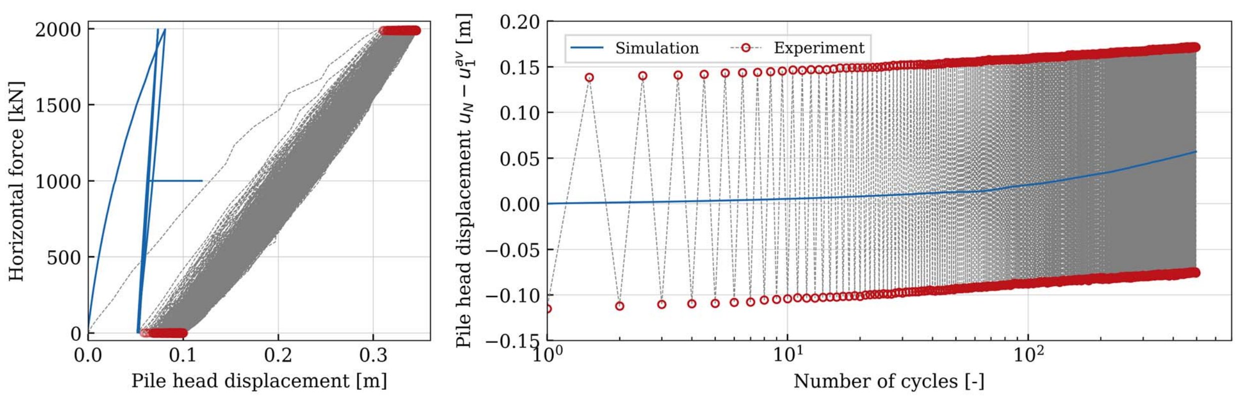

Fig. 10. Comparison between experiment and simulation of the cyclically loaded pile group: horizontal force at the pile head vs. pile head displacements (left); Pile head displacement vs. number of load cycles (right). Here, pile head refers to the load application point of the pile group (cf. Fig. 7). The red circles represent the pile head displacements measured at maximum and minimum load level of each cycle. The deformations from the cyclic loading are given relative to the deformation of the first irregular cycle measured at average load H_{av}.

As for the single pile, a linear trend of the accumulated pile head displacement in the semi-logarithmic representation is observed for the experiment. This linear increase in accumulated displacement is sufficiently well captured by the simulation (Fig. 10, right hand side). Hence, the HCA model prediction of the accumulated displacement of the pile group is satisfactory.

7. SUMMARY AND CONCLUSION

Centrifuge tests on monotonic and cyclic single pile tests and pile group tests are back-calculated using the FE code `numgeo`. A special calculation procedure is applied for the calculation of the soil deformations caused by cyclic loading combining a (conventional) hypoplastic constitutive model with Intergranular Strain extension and the high cycle accumulation (HCA) model. The material parameters of the hypoplastic model are calibrated based on the results of laboratory tests on the Silica Sand used in the centrifuge tests. An automatic calibration tool is used to simplify and speed up the calibration process. For the HCA model, a simplified calibration procedure was used based on index properties of the soil and three cyclic triaxial tests with varying strain amplitudes. The comparison of simulations and experiments showed that the applied numerical strategy is able to predict the accumulated pile head displacements of the single pile and the pile group sufficiently well. While the load deformation behavior of the single pile during the first cycles could be predicted quite well, discrepancies are noted for the pile group. In the latter case, the simulation of the first few load cycles were clearly stiffer than measured in the experiment and requires further investigation. While the accumulated deformations of the pile group were sufficiently well predicted, the accumulated deformations for the single pile were too large compared to the measurements. Better agreement between simulation and experiment are expected for a full calibration of the HCA model and the consideration of installation effects.

ACKNOWLEDGEMENTS

We thank Prof. Dr.-Ing. Oliver Reul and Dr.-Ing. Christoph Niemann for providing the data of the force-displacement curves of the centrifuge tests on the single pile and pile group. Furthermore, we thank Dr.-Ing. Felipe Prada, M.Sc. Frederik Koch and M.Sc. Alischa Hengelbrock for performing the laboratory tests on Silica Sand for the calibration of the material models at the facility of Chair of Soil Mechanics, Foundation Engineering and Environmental Geotechnics at Ruhr-University Bochum.

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