numgeo

A finite element code for geotechnical simulations

Tutorials
-
Series C: Muskat problem (Plaxis example)

www.numgeo.de
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History:

2022	Jan Machaček, Patrick Staubach	Initial version

`numgeo`: Tutorials, Series C: Infiltration of an initially dry sand column

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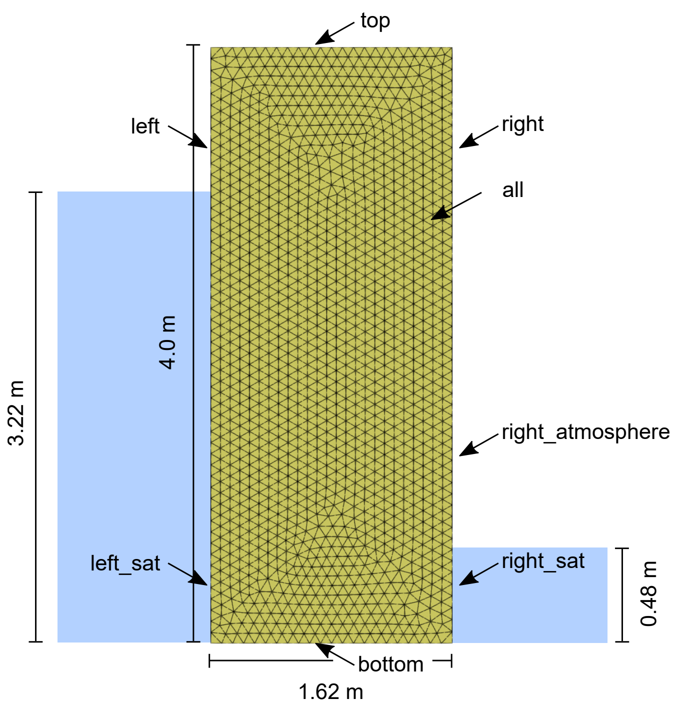
Contents

1 Introduction

Validation of element formulations (and implementations) for simulations of partially saturated problems is difficult due to lack of analytical solutions. For this reason, we take validate the implementations in numgeo based on a comparison with calculation results obtained with the widely used FE program Plaxis. The boundary value problem (BVP) considered for this purpose is taken from the validation example "Muskat Problem" of Plaxis (Vahid Galavi).

The BVP considers the unconfied flow of water in an earth dam. The soil dam has a heigt of 4 m, a width of 1.62 m and is displayed in Figure 1. The displacements are constrained at all nodes (only the flow of water is investigated in this example). The initial pore water pressure is assumed to be linearly distributed with the water table located at 0.48 m above the bottom boundary. Above 0.48 m the soil is initially partially saturated. During the analysis, the water table on left side of the dam is elevated up to a heigt of 3.22 m above the bottom boundary of the model. The distribution of the phreatic surface in the dam and the height of the seepage face (size saturataed area above the water table on the right-hand side of the dam) are the sought-after variables of this simulation.

Both the soil-water-retention curve and the dependence of the relative permeability on the effective degree of saturation are modelled using the well known van Genuchten model. The hydraulic conductivity K and the parameters of the van Genuchten model are chosen such as described in the Plaxis simulation: $K = 1.7604 \cdot 10^{-6}$ m/s, $n^{vG} = 1.377$ and $\alpha^{vG} = 0.383$. The residual degree of saturation is $S^{res} = 0.063$. No information about the bulk modulus of the pore water K^w is provided, thus the bulk modulus is assumed to correspond to the one of pure water $K^w = 2.2 \cdot 10^6$ kPa. The initial void ratio is $e_0 = 0.5$



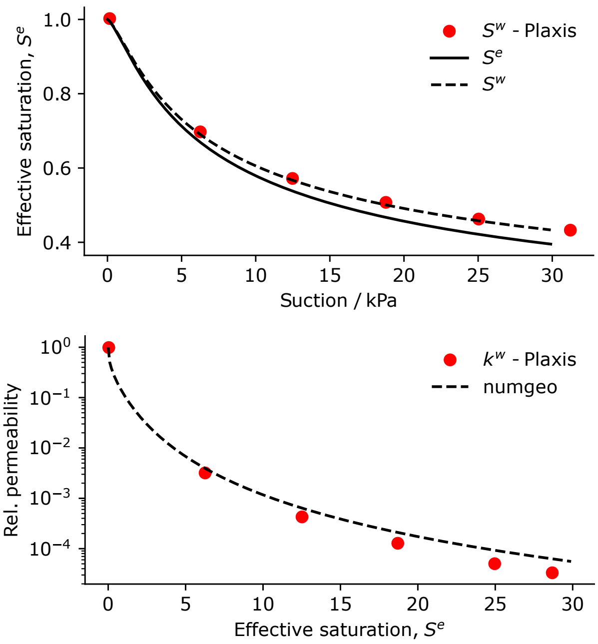


Figure 1: Left: finite element model of the BVP, Middle: initial distribution of pore water pressure, Right: comparison of soil-water-retention curve and relative permeabiltiy used in Plaxis and numgeo.

2 Numerical simulation

2.1 Material

For the solid a linear elastic constitutive model is chosen. As no soil deformation is considered in this simulation (an neither was observed in the experiment) this choice is completely arbitrary. The Young's modulus is 10^3 kPa and the Poisson's ratio 0.3.

Note that numgeo requires the prescription of the permeability K^s of the solid and the dynamic viscosity of the pore fluids μ^f instead of the hydraulic conductivity K^f, which are related as follows:

$$K^f = \frac{K^s \gamma^f}{\mu^f} \tag{1}$$

Therein, γ^f and μ^f are the specific weight and the dynamic viscosity of the fluid f, respectively.
The dynamic viscosity of pore water is $\mu^w = 10^{-6}$ m·s and of the pore air $\mu^a = 10^{-8}$ m·s. Assuming a specific weight of 10 kN/m^3 for the pore water, the permeability of the soil is calculated to $1.157407 \cdot 10^{-12}$ m^2 (corresponding to 1 m/day).

The corresponding input commands are given in Listing 1.

```
*Material , name = elastic , phases = 3
*Mechanical = linear_elasticity
50d3 , 0.3
*Density
2.65 , 1.0 , 0.0015
*Bulk modulus
2.2d6 , 100.
*Dynamic viscosity
1d−6, 1d−8
*Permeability = isotropic
1.157407d−12
*Hydraulic = van Genuchten , Swr=0.063**0.4
0.383 , 1.377
*Relative permeability = van Genuchten
1d−6, 1d−6, 1.377
*Bishop effective stress = Crude−Switch
```

Listing 1: Definition of the material

2.2 Geometry and boundary conditions

For the simulation we model the dam as a planar (2D) situation. The entire model consists of one part named "Soil". On this part a total of 8 node sets and one element set were defined:

- top (Soil.top)
- bottom (Soil.bottom)
- left (Soil.left)
- left (Soil.left_sat)
- right (Soil.right)
- right (Soil.right_sat)
- right (Soil.right_atmosphere)
- all (Soil.all, element and node set)

The finite element mesh was created using the open-source software Salome [Ribes and Caremoli, 2007] and the numgeo-Python API. The dam is discretised with 6-noded triangular elements (quadratic interpolation). The nodal distance is approximately 0.05 m. For this simulation, changes in pore air pressure are judged as negligible, thus elements based on reduced set of governing equations are used - namely the up-formulation. These elements consider negative pore water pressures as suction $s = -p^w$ (instead of $s = p^a - p^w$). The geometry as well as some of the defined node sets are displayed in Fig. 1.

The input files as well as the Salome model (*.hdf) are included in the enclosed data.

2.3 Initial conditions

For the initial pore water pressure the water level is assumed to be located at a height of 0.48 m above the bottom of the model. This results in a linear distribution of pore water pressure taking values of $p_0^w = 4.8$ kPa at the bottom, $p_0^w = 0$ kPa at 0.48 m and $p_0^w = -35.2$ kPa at the top of the dam. The initial void ratio is $e_0 = 0.5$. The corresponding input commands are:

```
*Initial conditions, type=stress, geostatic
Soil.all, 0.0, -42, 4.0, 0., 0.5, 0.5

*initial conditions, type=void ratio, default
Soil.all, 0.5

*initial conditions, type=pore water pressure, default
Soil.all, 0.0d0, 4.8d0, 4.d0, -35.2d0
```

Listing 2: Definition of initial conditions

2.4 Calculation stages

The simulation is divided into 2 steps in total: one Geostatic step and one transient step.

Geostatic step

During the Geostatic step, the self weight of the soil (grains and pore water) is applied without generating any deformation. As stated previously, no deformation of the soil skeleton is expected. We therefore constrain the displacements of all nodes in $x1$ and $x2$ direction. In addition, we use boundary conditions to prescribe the pore water pressure for each node. As in the initial conditions, the pore water pressure is prescribed using the `type=hydrostatic` option which generates the desired linearly varying distribution as described in Section 2.3. The corresponding input commands are given in Listing 3.

```
*Step, name=Geostatic, inc=1, maxiter=100
*Geostatic

*Body force, instant
Soil.all, grav, 10.0, 0., -1, 0.

*Boundary
Soil.all, u1, 0.
Soil.all, u2, 0.
Soil.bottom, pw, 4.8d0
Soil.top, pw, -30.2d0

*Boundary, type=hydrostatic
Soil.right_sat, pw, 10.0, 0.48

*Output, field, vtk, ascii
*Frequency = 1
*Element, elset = Soil.all
S, sat_eff, void, sat
*Node, nset = Soil.all
pw, sat_eff, void, sat

*End Step
```

Listing 3: Definition of the Geostatic step

Transient step

During the transient step we simulate the water supply at the left side of the dam (node set `soil.right_sat`). This is done by prescribing the pore water pressure at the corresponding nodes using a user defined subroutine (using the `*UBoundary` keyword). The corresponding code is provided in Listing 4.

```
subroutine user_boundary_conditions(dof,inode,istep,time,coords,bc_value) &
      bind(c,name='user_boundary_conditions')
      use, intrinsic :: iso_c_binding
      implicit none
```

```
      character(c_char)          , intent(in)    :: dof
      integer(c_int)             , intent(in)    :: inode
      integer(c_int)             , intent(in)    :: istep
      real(c_double)             , intent(in)    :: time
      real(c_double), dimension(3), intent(in)    :: coords
      real(c_double), dimension(3), intent(inout) :: bc_value

      real(c_double) :: gammaW, pw_0, final_pw, rising_time, m

  gammaW = 10.0d0
  rising_time = 0.1d0 * 24.0d0 * 60.0d0 * 60.0d0
  if(istep == 2) then
    pw_0 = - (coords(2)-0.48)*gammaW
    final_pw = (3.22d0-coords(2)) * gammaW
    if (time < rising_time) then
      m = (final_pw-pw_0)/rising_time
      bc_value(1) = pw_0 + m*time
    else
      bc_value(1) = final_pw
    end if
  endif

end subroutine user_boundary_conditions
```

Listing 4: User defined subroutine to prescribe the pore water pressure at the upstream slope of the dam.

The total step time is 223136 seconds. Due to the strong nonlinearities resulting from the saturation-suction relation and the relative permeability function, we limit the maximum allowed time increment size to 2000 seconds. For the rise of the water level a time of 2.4 hours is assumed. The corresponding input commands are given in Listing 5.

```
*Step, name=Saturation, inc=1000000, maxiter=50

*Transient
0.01, 223136, 0.01, 2000

*Body force, instant
Soil.all, grav, 10.0, 0., -1, 0.

*Boundary
Soil.all, u1, 0.
Soil.all, u2, 0.

*Boundary, type=hydrostatic
Soil.right_sat, pw, 10.0, 0.48

*UBoundary
Soil.left_sat, pw, 1.

*DSload, instant
surf_right, drainage-w, 100d0

*controls, global, deactivate
*controls, pw, activate

*End Step
```

Listing 5: Definition of the transient step

2.5 Results

Figure 2 presents the comparison of the simulation results obtained with numgeo and the results presented in the validation example of Plaxis by means of the distribution of pore water pressure in steady state conditions. It can be seen, that the pressure distributions obtained by Plaxis and numgeo are in good agreement. Following the phreatic surface (line/surface at which $p^w = 0$ kPa holds) the seepage face s can be identified. The calculated seepage face by means of FEM (numgeo and Plaxis) fit reasonably well to the reference analytical solution (Muskat).

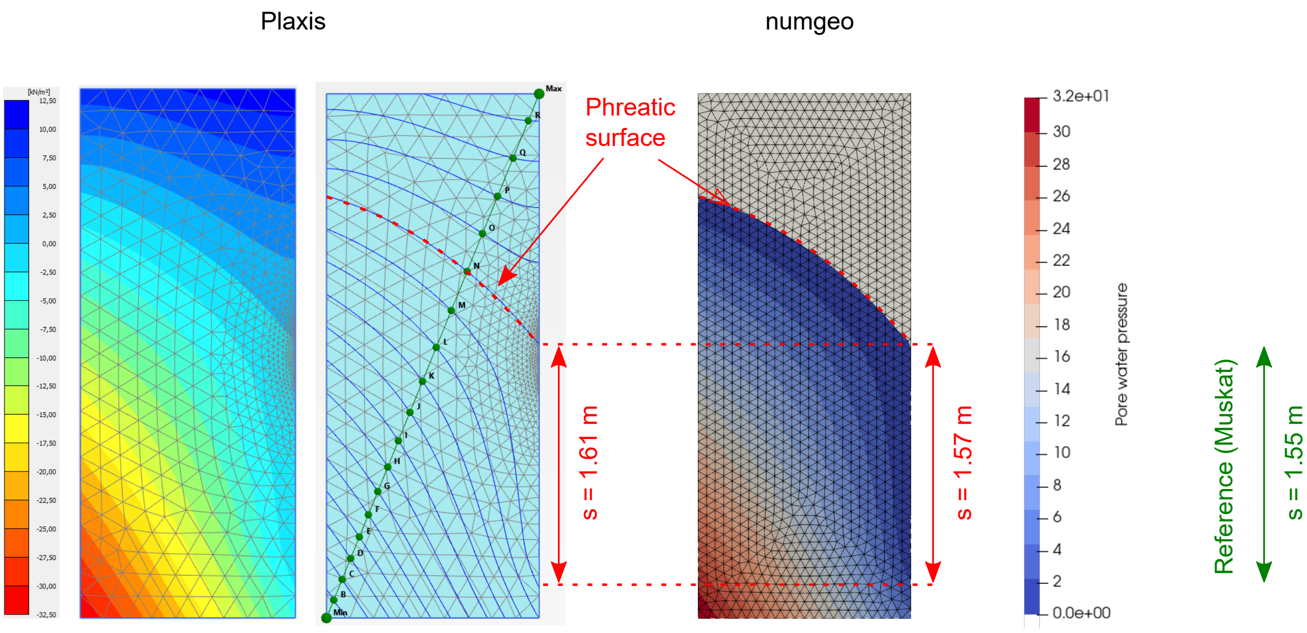


Figure 2: Distribution of pore water pressure obtained by Plaxis (left) and numgeo (right) as well as height of the seepage face s (m).

References

A. Ribes and C. Caremoli. Salome platform component model for numerical simulation. In *31st annual international computer software and applications conference (COMPSAC 2007)*, volume 2, pages 553–564. IEEE, 2007.