A FINITE ELEMENT CODE FOR GEOTECHNICAL SIMULATIONS

TUTORIALS
-
SERIES C: INFILTRATION OF A HOMOGENEOUS EARTH DAM

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History:

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Contents

1 Introduction

The series C tutorials aim to introduce the user to the simulation of unsaturated (two-phase) flow in a porous medium. The first tutorial of this series dealt with the back-calculation of a laboratory test performed by Skaggs et al. [1970]. In this tutorial, the application of the two-phase formulation with a variable degree of water saturation is demonstrated by the numerical simulation of water flow through an earth dam depicted in Fig. 1. A similar boundary value problem was used by Oettl et al. [2004] to demonstrate a possible application of multi phase models in geotechnical engineering. The dam is resting on an impervious base layer. The cross-section of the dam has a width of 56.0 m at the base which is reduced to 4.0 m at the crest. The height of the dam is 12.0 m. The drainage of the leakage occurring through the dam is prevented with a drain with a length of 12.0 m at the base of the downstream slope. As in classical seepage analysis, a rigid soil skeleton is assumed and only the flow problem is investigated.

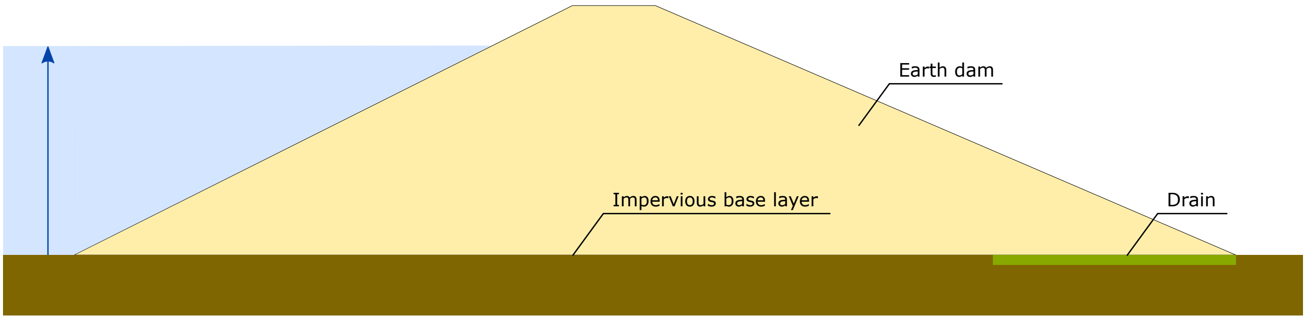

Figure 1: Homogeneous earth dam.

As a consequence of the failure of a nearby dam, the water level at the upstream slope (which was initially dry) raises in two days by 10 m, as stretched out in Fig. 1. We are interested in the evolution of the phreatic surface following this event.

2 Numerical simulation

2.1 Constitutive models

The van Genuchten model [van Genuchten, 1980] is chosen as the saturation-suction relation and the relative permeability function. The relations for both the capillary stress and the relative permeability are depicted in Fig. 2. The reasoning behind plotting the derivative of the effective degree of saturation (Fig. 2 middle) will be explained in Section 2.4. The van Genuchten material parameters are $n^{vG} = 2.5$ and $\alpha^{vG} = 3$. The residual degree of saturation is $S^{wr} = 0.25$.

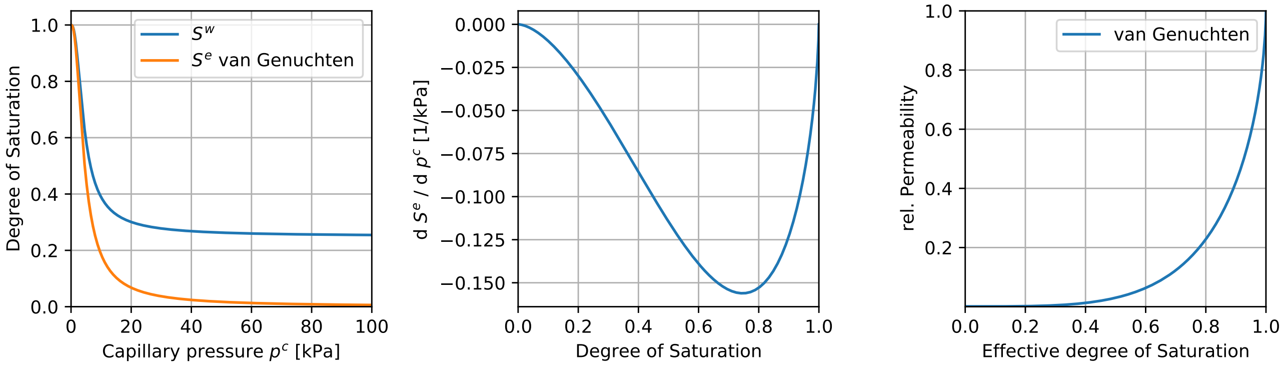

Figure 2: Saturation-suction relation (left) and relative permeability function (right).

For the solid a linear elastic constitutive model is chosen. As no soil deformation is considered in this simulation (an neither was observed in the experiment) this choice is completely arbitrary. The Young's modulus is 10^3 kPa and the Poisson's ratio 0.25.

A permeability of the soil of $1 \cdot 10^{-14}$ and a dynamic viscosity of the pore water of $\mu^w = 10^{-6}$ are chosen, which corresponds to a hydraulic conductivity of $1 \cdot 10^{-7}$. The specific weight of the pore water is $\gamma^w = \rho^w \cdot g = 1.0 \cdot 10.0 = 10$ kN/m^3. The Bulk modulus of pore water and pore air are 2.0 GPa and 101 kPa, respectively.

The corresponding input commands are given in Listing 1.

```
*Material, name = mat1, phases = 3
*Mechanical = linear_elasticity
10.d3, 0.25
*Density
2.6, 1., 0.01
*Permeability = isotropic
1.0d-14
*Dynamic viscosity
1.0d-6, 1.d-8
*Relative permeability = van Genuchten
2.5
*Hydraulic = van Genuchten, swr = 0.25
3.0, 2.5
*Bulk modulus
2.d6, 101.
```

Listing 1: Definition of the Geostatic step

2.2 Geometry and boundary conditions

The finite element mesh was created using the open-source software Salome [Ribes and Caremoli, 2007] and the numgeo-Python API. The dam is discretised with 8-noded rectangular elements and 6-noded triangular elements (quadratic interpolation). The nodal distance is approximately 0.15 m. The geometry as well as some of the defined node sets are displayed in Fig. 3.

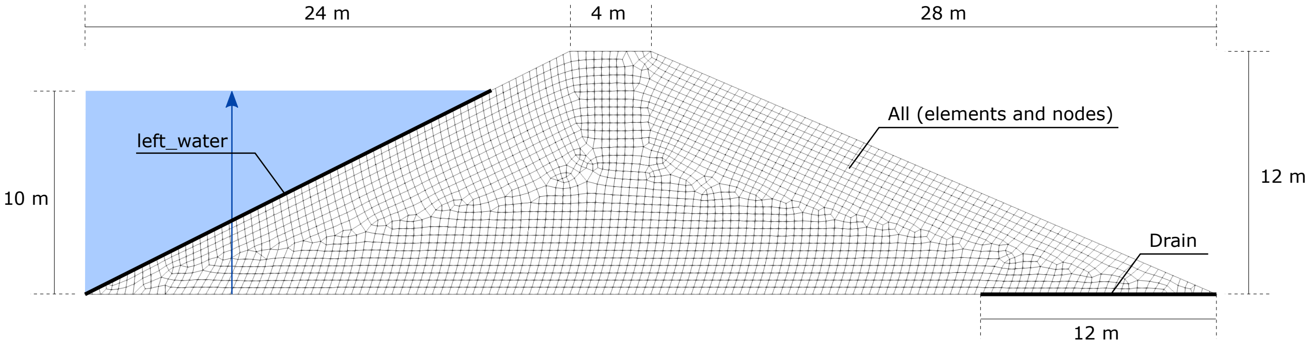

Figure 3: Finite element model of the homogeneous earth dam.

Neither the impervious base layer nor the drain are explicitely discretised in the numerical model. Obviously, this implies the assumption that the drain was sufficiently dimensioned to discharge the accruing water pressure-free. The entire model consists of one part named "Dam". On this part a total of 5 node sets and one element set were defined:

- left_water (Dam.left_water)
- drain (Dam.drain)
- all (Dam.all, element and node set)

For this simulation, changes in pore air pressure are judged as negligible, thus elements based on reduced set of governing equations are used - namely the up-formulation. These elements consider negative pore water pressures as suction $s = -p^w$ (instead of $s = p^a - p^w$). The input commands for the selected elements are given in Listing 2.

```
*Element , Type = u8p4
...
*Element , Type = u6p3
...
```

Listing 2: Assigned element types

2.3 Initial conditions

The initial conditions for pore water pressure must have a gradient that is equal to the specific weight of the fluid so that, according to Darcy's law, there is no initial flow. For this purpose we assume that the water table before the event is located directly below the earth dam, thus the initial pore water pressures vary linearly from 0 kPa at the bottom of the dam to -120 kPa at the top of the dam, see Fig. 4. The resulting initial saturation is as well displayed in Fig. 4. The initial void ratio is $e_0 = 0.5$. The initial conditions are displayed in Fig. 4).

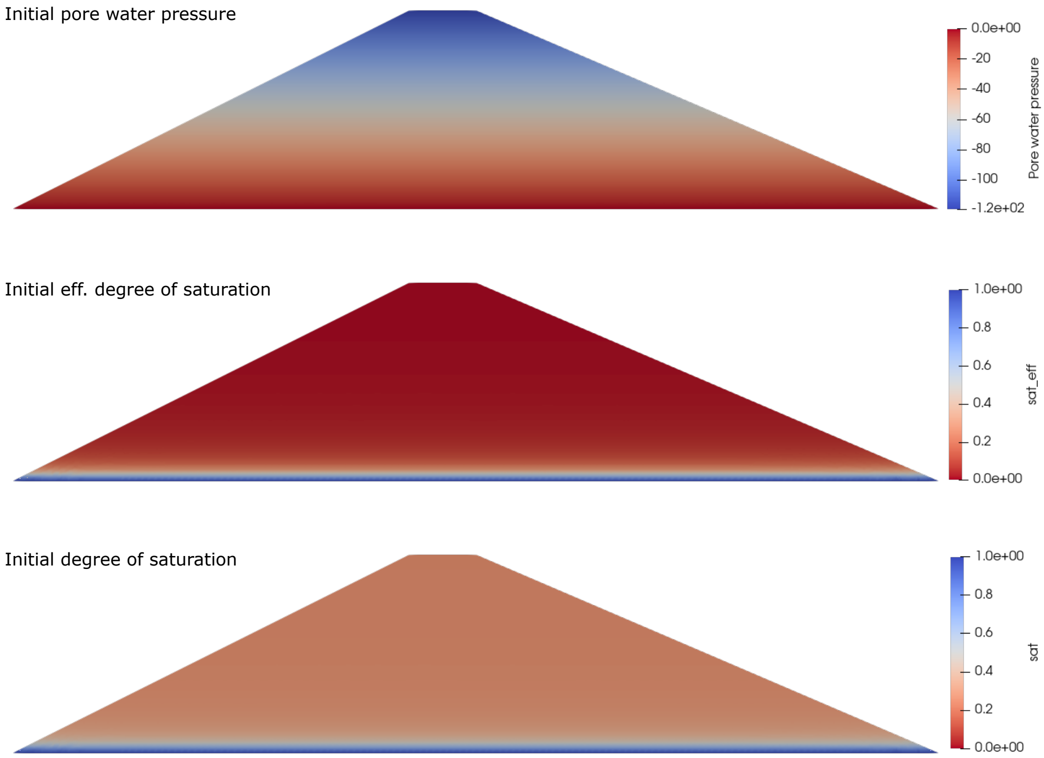

Figure 4: Initial pore water pressure (top) and initial effective degree of saturation (right).

The corresponding input commands are given in Listing 3.

```
*Initial conditions, type = stress, geostatic
dam.all, 12., 0., 0., -209., 0.5, 0.5
**
*Initial conditions, type = pore water pressure, default
dam.all, 0.0, 0., 12., -120.
**
*Initial conditions, type = void ratio, default
dam.all, 0.5
```

Listing 3: Initial conditions

2.4 Critical time step

Before moving to the definition of the calculation stages we investigate possible restrictions in the choice of the time step in the transient analysis. Especially the choice of initial time increment in a transient partially saturated flow problem is important to avoid spurious solution oscillations. These non-physical oscillations may cause problems if pressure-sensitive constitutive models are used to model the porous medium and may lead to convergence difficulties in partially saturated analyses. A criterion for the minimum usable time increment in partial-saturation conditions is:

$$\Delta t \geq \frac{\gamma^w n^e}{C k^w k} \frac{\mathrm{d}S^e}{\mathrm{d}p^c} (\Delta l)^2 \tag{1}$$

Where C is a constant ranging from $C = 6$ [Hibbitt et al., 1997] to $C = 20$ [Thomas and Zhou, 1997] for quadratic interpolated elements. $\frac{\mathrm{d}S^e}{\mathrm{d}p^c}$ is the slope of the capillary pressure - saturation relation (see Fig. 2) and Δl is the average nodal distance in the model. k^w is the relative permeability with respect to the pore water and $k = \min K_{ii}$ is the minimum component of the hydraulic conductivity tensor of the material. It should now be realized that this criterion is not exact. It only provides an estimate for a minimum time step size (or maximum element sizes). For the present simulation, we evaluated Eq. (1). The results are given in Fig. 5. Given that most areas of the earth dam have an initial effective degree of saturation of $S_0^e \leq 0.05$, we estimate an initial time step of $\Delta t_0 \geq 2500$ seconds based on Fig. 5.

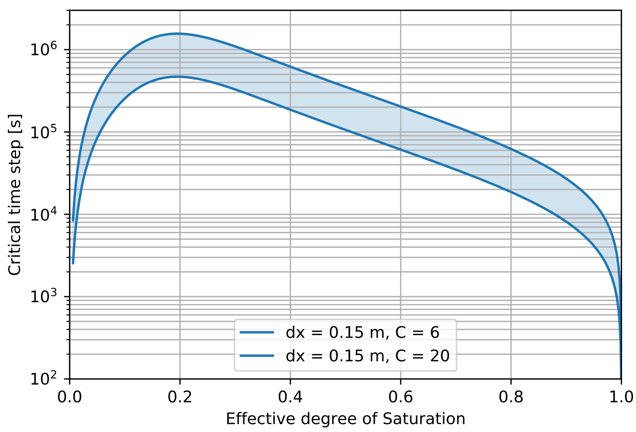

Figure 5: Critical time step as function of the effective degree of saturation.

2.5 Calculation stages

The simulation is divided into 2 steps in total: one Geostatic step and one transient step.

Geostatic step

During the Geostatic step, the self weight of the soil (grains and pore water) is applied without generating any deformation. As stated previously, no deformation of the soil skeleton is expected. We therefore constrain the displacements of all nodes in x_1 and x_2 direction. In addition, we use boundary conditions to prescribe the pore water pressure for each node. As in the initial conditions,the pore water pressure must have a gradient that is equal to the specific weight of the pore water. This is achieved using the `type=hydrostatic` keyword in the `*Boundary` command. The corresponding input commands are given in Listing 4.

```
*Step, name = step_1, inc = 1
*Geostatic
**
*Solver, mumps
**
*Body force, instant
dam.all, grav, 10., 0, -1, 0
**
*Boundary
dam.all, u1, 0.
dam.all, u2, 0.
**
*BOUNDARY, type=hydrostatic
dam.all,pw,10.0d0,0.001d0
**
*Output, field, vtk, ascii
*frequency=1
*Node output, nset=dam.all
U, pw
*Element output, elset=dam.all
S, E, sat_eff, darcy_w1, darcy_w2, sat
```

Listing 4: Definition of the Geostatic step

Transient step

The water supply at the top of the upstream slope of the earth dam is simulated using a transient step type. The water level in the reservoir is raised during a period of two days. Afterwards, the water level is held constant in the course of the calculation. The raise of the water level is simulated by prescibing a pore water pressure equivalent to the hydrostatic water pressure acting on the upstream slope of the dam. Since the pore water pressures along the upstream slope not only vary with time but also depending on ther location in space, we use an user defined routine to prescibe the pore water pressure. The code used for this is given in Listing 5. The usage of the user defined boundary condition is invoked by the `numgeo` keyword `*UBoundary` as can be seen from Listing 6. The pore water pressure at the nodes of the drain are held constant ($p^w = 0$). As mentioned in Section 2.2 this implies that drain is designed such that the accruing water can be discharged pressure-free.

```
subroutine user_boundary_conditions(dof,inode,istep,time,coords,bc_value) &
  bind(c,name='user_boundary_conditions')
  use, intrinsic :: iso_c_binding
  implicit none
  character(c_char)              , intent(in)    :: dof
  integer(c_int)                 , intent(in)    :: inode
  integer(c_int)                 , intent(in)    :: istep
  real(c_double)                 , intent(in)    :: time
  real(c_double), dimension(3), intent(in)    :: coords
  real(c_double), dimension(3), intent(inout) :: bc_value
  real(c_double) :: gammaW, initial_suction, final_pw, rising_time, m

  gammaW = 10.0d0
  rising_time = 2.0d0 * 24.0d0 * 60.0d0 * 60.0d0

  if(istep == 2) then
    initial_suction = - coords(2)*gammaW
    final_pw = (10.0d0-coords(2)) * gammaW
    if (time < rising_time) then
```

```
      m = (final_pw-initial_suction)/rising_time
      bc_value(1) = initial_suction + m*time
    else
      bc_value(1) = final_pw
    end if
  endif

end subroutine user_boundary_conditions
```

Listing 5: User defined subroutine to prescribe the pore water pressure at the upstream slope of the dam.

The check on pore pressure changes is relaxed using solution controls. The analysis can also be done with stricter convergence criteria, but numgeo iterates a lot more without any gain in solution accuracy. The corresponding input commands are given in Listing 6.

```
*Step, name = raise_waterlevel, inc = 1000000
*Transient
3000, 648.0d6, 0.0001, 2.16d6
**
*Solver, mumps
**
*Body force, instant
dam.all, grav, 10., 0, -1, 0
**
*Boundary
dam.all, u1, 0.
dam.all, u2, 0.
dam.drain, pw, 0.
**
*UBoundary
dam.left_water,pw, 1.
**
*Output, field, vtk, ascii
*frequency=20
*Node output, nset=dam.all
U, pw
*Element output, elset=dam.all
S, E, sat_eff, darcy_w1, darcy_w2, sat
**
*Controls, pw, modify
0.05,0.025,0.025,1E-6,1E-9
**
*End Step
```

Listing 6: Definition of the Transient step

2.6 Results

For the present simulation, there are two methods to evaluate the location of the so-called phreatic surface. The phreatic surface is characterized by both, zero pore water pressure and a degree of saturation equal to one. Based on the first criterion, we can evaluate the location of the phreatic surface using the post processing software Paraview [Ahrens et al., 2005]. Using the *Contour* filter with a *value range* of zero for the isolines, we obtain the results presented in Fig. 6. The typical S-shaped curve indicating the position of a zero pore water pressure, stretches from the position of the water level (left) to the upper end of the drain (right). In the region above this curve the capillary pressure (negative pore water pressure) according to the initial condition prevails.

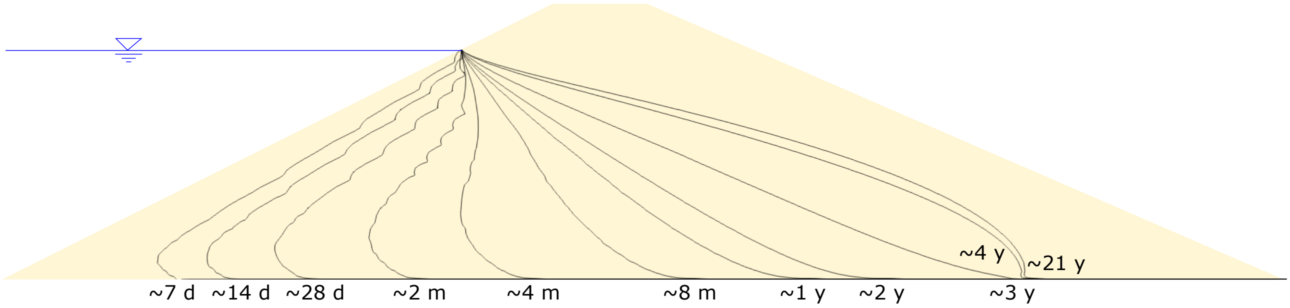

Figure 6: Location of the phreatic surface evaluated on the zero pore pressure criterion for different times of the simulation.

The second method for evaluating the location of the phreatic surface is by plotting the distribution of the (effective) degree of saturation. Remember at fully saturated states, $S^e = S^w$, thus it does not matter if S^e or S^w is plotted. To better determine the exact location of the phreatic surface, we only plot degrees of saturation exceeding 0.9. Figure 7 shows the distribution of the effective degree of saturation after approx. 4 months and 21 years of infiltration. The arrows indicate areas with high Darcy velocities (the larger the arrow, the larger the velocity) and the flow direction.

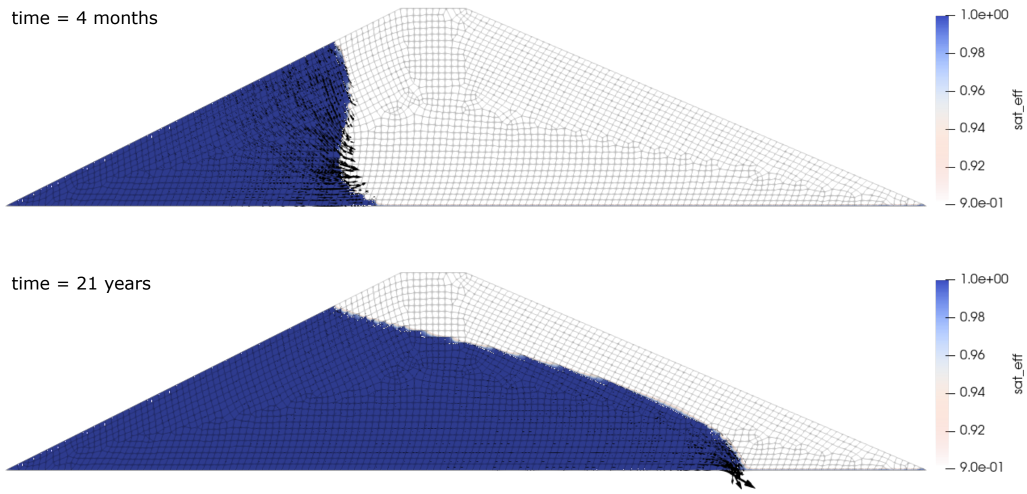

Figure 7: Location of the phreatic surface evaluated on the zero pore pressure criterion for different times of the simulation.

Comparing Fig. 6 and Fig. 7 we see that both methods result in the same location of the phreatic surface.

A Model creation with Salome

B Post-processing with Paraview