numgeo

A finite element code for geotechnical simulations

Tutorials
-
Series C: Infiltration of an initially dry sand column

www.numgeo.de
September 29, 2022

TECHNISCHE UNIVERSITÄT DARMSTADT
RUB
Bauhaus-Universität Weimar

History:

2021	Jan Machaček, Patrick Staubach	Initial version

`numgeo`: Tutorials, Series C: Infiltration of an initially dry sand column

www.numgeo.de
September 29, 2022

Contents

1 Introduction

The series C tutorials aim to introduce the user to the simulation of unsaturated (two-phase) flow in a porous medium. As it is hard to derive analytical solutions for the unsaturated flow by means of a validation example, we perform a back analysis of a laboratory test performed by Skaggs et al. [1970].

Skaggs et al. performed experiments to obtain the hydraulic conductivity of unsaturated soil in a soil column. These experiment include a series of infiltration tests of water in initially dry soil. The soil column was 8.75 cm long, 8.75 cm wide and 61 cm high. The water was applied to the soil surface through an applicator plate made of perforated Plexiglas. Prior to the experiments the saturation-suction relation and the hydraulic conductivity at saturated states were determined. The pressure head during the saturation of the sand column was 0.75 cm (or 0.075 kPa) and held constant throughout the test. During the test the infiltration rate and the location of the wetting front were measured. The function relating the relative permeability to the degree of saturation (water content) was back-calculated from the experiments.

The saturated hydraulic conductivity was 2.67 cm/hr ($7.42 \cdot 10^{-6}$ m/s). The saturation-suction relation and the relative permeability are presented in Fig. 1.

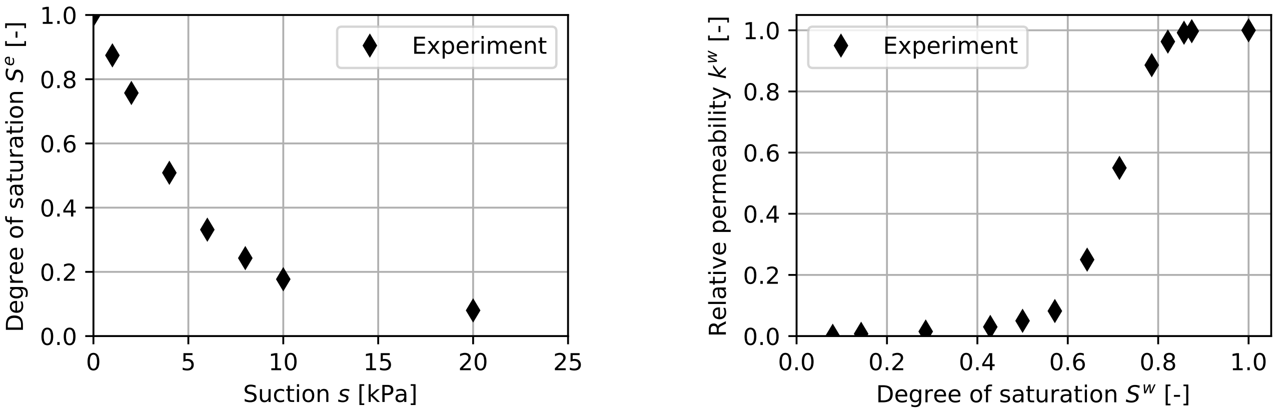

Figure 1: Experimental saturation-suction relation (left) and relative permeability function (right). Data taken from Skaggs et al. [1970].

The initial density of the soil column was determined to $\rho^d = 1.72$ g/cm^3. Assuming a grain density of $\rho^s = 2.62$ g/cm^3 we estimate an initial void ratio of $e_0 \approx 0.5$.

2 Numerical simulation

2.1 Constitutive models

The material parameters for the saturation.suction relation and the relative permeability function are calibrated based on the experimental findings of Skaggs et al. [1970]. The saturation-suction relation was well reproduced by the van Genuchten constitutive model [van Genuchten, 1980]. The van Genuchten parameter are $\alpha^{vG} = 3.2$ and $n^{vG} = 3.0$. The comparison of the constitutive model and the experiment is depicted in Fig. 2 (left).

For the relative permeability function neither the Brooks & Corey model, the van Genuchten model nor the Nguyen model were able to reproduce the observed behaviour. A good agreement with the experiment was found using the Logistic-fit curve fitting model:

$$k^\beta = \frac{1 - k^\beta_{min}}{1 + e^{\chi^\beta (S^e - \zeta)}} + k^\beta_{min} \quad \text{with:} \quad \chi^w = -\chi \quad \text{and} \quad \chi^a = \chi \tag{1}$$

The parameter are $k^w_{min} = k^a_{min} = 0.01$, $\chi = 20$ and $\zeta = 0.7$. A comparison of simulated relative permeabilities with the experiment is provided in Fig. 2 (right).
For the solid a linear elastic constitutive model is chosen. As no soil deformation is considered in this simulation (an neither was observed in the experiment) this choice is completely arbitrary. The Young's modulus is 10^3 kPa

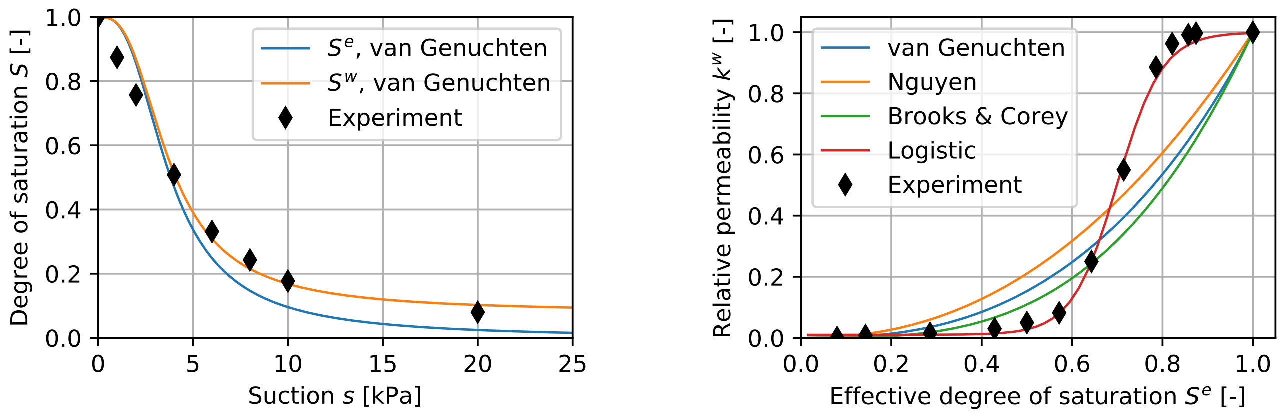

Figure 2: Comparison of simulated and experimental saturation-suction relation (left) and relative permeability function (right). Data taken from Skaggs et al. [1970].

and the Poisson's ratio 0.25.

The experimentally determined hydraulic conductivity of $7.42 \cdot 10^{-6}$ corresponds to a permeability of the soil of $7.42 \cdot 10^{-13}$ and a dynamic viscosity of the pore water of $\mu^w = 10^{-6}$. The specific weight of the pore water is $\gamma^w = \rho^w \cdot g = 1.0 \cdot 10.0 = 10$ kN/m^3. The Bulk modulus of pore water and pore air are 2.0 GPa and 101 kPa, respectively.

The corresponding input commands are given in Listing 1.

```
*Material, name = mat1, phases = 3
*Mechanical = linear_elasticity
10.d3, 0.25
*Density
2.6, 1., 0.01
*Permeability = isotropic
7.42d-13
*Dynamic viscosity
1.0d-6, 1.d-8
*Relative permeability = Logistic-fit
0.01, 0.01, 20, 0.7
*Hydraulic = van Genuchten, swr = 0.08
3.2, 3.0
*Bulk modulus
2.d6, 101.
```

Listing 1: Definition of the material

2.2 Geometry and boundary conditions

For the back calculation of the experiments we simplify the geometry of the soil column as a planar (2D) situation. As in the experiment, the soil column is 8.75 cm long, 8.75 cm wide and 61 cm high. The entire model consists of one part named "Soil". On this part a total of 5 node sets and one element set were defined:

- top (Soil.top)
- bottom (Soil.bottom)
- left (Soil.left)
- right (Soil.right)
- all (Soil.all, element and node set)

The finite element mesh was created using the open-source software Salome [Ribes and Caremoli, 2007] and the numgeo-Python API. The column is discretised with 8-noded rectangular elements (quadratic interpolation). The nodal distance is approximately 0.01 m. For this simulation, changes in pore air pressure are judged as negligible, thus elements based on reduced set of governing equations are used - namely the **up**-formulation. These elements consider negative pore water pressures as suction $s = -p^w$ (instead of $s = p^a - p^w$). The geometry as well as some of the defined node sets are displayed in Fig. 3.

2.3 Initial conditions

The initial condition for saturation is 8% throughout the column. The initial conditions for pore water pressure must have a gradient that is equal to the specific weight of the fluid so that, according to Darcy's law, there is no initial flow. For this purpose we assume that the initial pore water pressures vary linearly from -6.0 kPa at the bottom of the column to -12.1 kPa at the top of the column. These initial conditions satisfy the pore pressure/saturation relationship displayed in Fig. 2. The initial void ratio is $e_0 = 0.5$. The initial conditions are displayed in Fig. 3 (right).

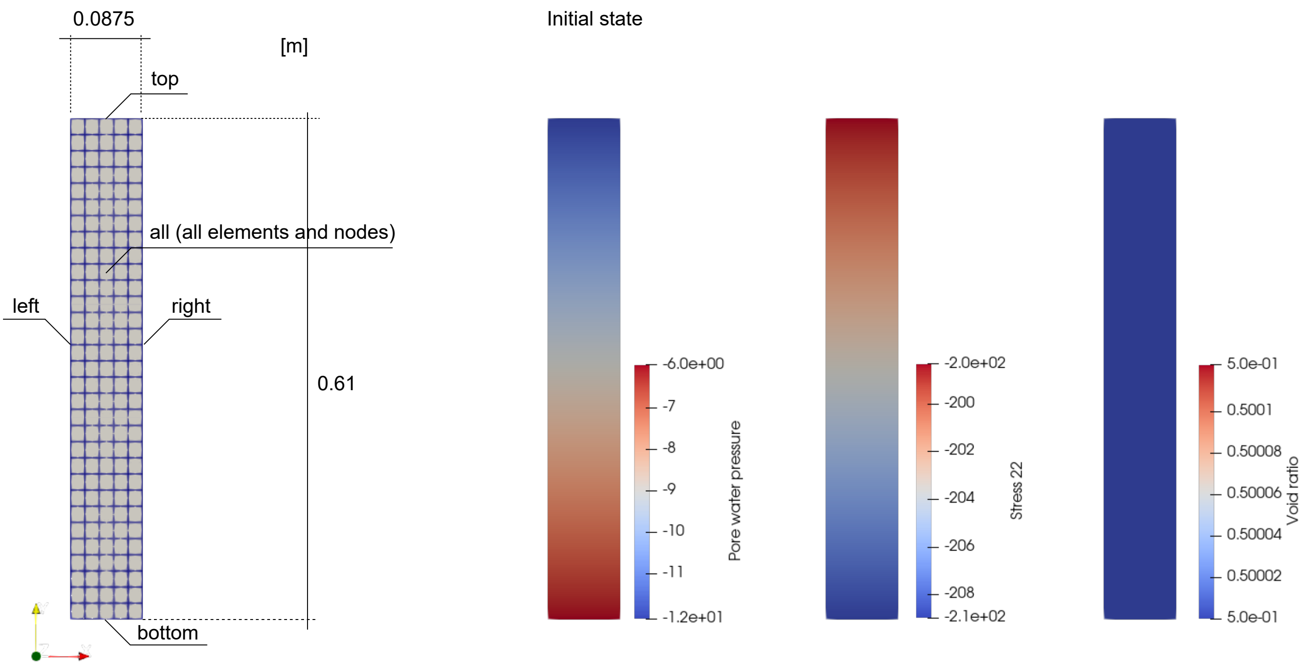

Figure 3: Finite element model (left) and initial state (right).

The corresponding input commands are given in Listing 2.

```
*Initial conditions, type = stress, geostatic
Soil.all, 12., 0., 0., -209., 0.5, 0.5
**
*Initial conditions, type = pore water pressure, default
Soil.all, 0.0, -6.0, 0.61, -12.1
**
*Initial conditions, type = void ratio, default
Soil.all, 0.50
```

Listing 2: Definition of initial conditions

2.4 Calculation stages

The simulation is divided into 2 steps in total: one Geostatic step and one transient step.

Geostatic step

During the Geostatic step, the self weight of the soil (grains and pore water) is applied without generating any deformation. As stated previously, no deformation of the soil skeleton is expected. We therefore constrain the displacements of all nodes in $x1$ and $x2$ direction. In addition, we use boundary conditions to prescribe the pore water pressure for each node. As in the initial conditions,the pore water pressure must have a gradient that is equal to the specific weight of the pore water. This is achieved using the `type=hydrostatic` keyword in the `*Boundary` command. The corresponding input commands are given in Listing 3.

```
*Step, name = step_1, inc = 1
*Geostatic
**
*Solver, mumps
**
*Body force, instant
Soil.all, grav, 10., 0, -1, 0
**
*Boundary
Soil.all, u1, 0.
Soil.all, u2, 0.
**
*BOUNDARY, type=hydrostatic
Soil.all,pw,10.0d0,-0.60d0
**
*Output, field, vtk, ascii
*frequency=1
*Node output, nset=dam.all
U, pw
*Element output, elset=dam.all
S, E, sat_eff, darcy_w1, darcy_w2, sat, void
**
**
*End Step
```

Listing 3: Definition of the Geostatic step

Transient step

During the transient step we simulate the water supply at the top of the soil column. This is done by prescribing the pore water pressure at the top nodes. The value is set to 0.075 kPa, which corresponds to a pressure head of 0.75 cm. The total step time is 5600 seconds. Due to the stron nonlinearities resulting from the saturation-suction relation and the relative permeability function, we limit the maximum allowed time increment size to 50 seconds. No ramp is used to increase the pore water pressure from its initial value of -12.1 kPa (`*Boundary, instant`). The check on pore pressure changes is relaxed using solution controls. The analysis can also be done with stricter convergence criteria, but `numgeo` iterates a lot more without any gain in solution accuracy. The corresponding input commands are given in Listing 4.

```
*Step, name = infiltration, inc = 1000000
*Transient
5, 5600, 0.0001, 50
**
*Solver, mumps
**
*Body force, instant
Soil.all, grav, 10., 0, -1, 0
**
*Boundary
Soil.all, u1, 0.
Soil.all, u2, 0.
**
*BOUNDARY, instant
Soil.top,pw,0.075
**
*Output, field, vtk, ascii
*frequency=1
*Node output, nset=Soil.all
U, pw
*Element output, elset=Soil.all
S, E, sat_eff, darcy_w1, darcy_w2, sat
**
*Controls, pw, modify
0.05,0.025,0.025,1E-6,1E-9
**
*End Step
```

Listing 4: Definition of the transient step

2.5 Results

Figure 4 presents the measured data and the simulation of the evolution of the wetting front. The figure shows good agreement between the simulation with `numgeo` and the measured data. We consider the wetting front where the degree of saturation exceeds $S^w > 0.75$.

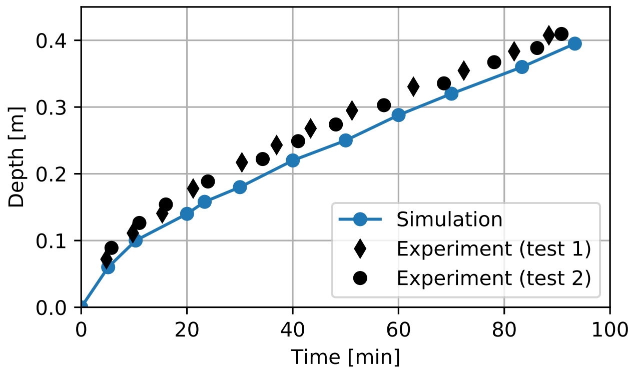

Figure 4: Measured and simulated movement of the wetting front.