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Automatic parameter calibration of two sophisticated soil models based on monotonic and cyclic tests on sand

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ABSTRACT: Simulating the highly nonlinear behaviour of soils under undrained cyclic loading, requires advanced constitutive models. Determining the parameters of these models is often time-consuming and requires a high level of expertise. In order to simplify and speed up the calibration process and increase the accessibility of advanced soil models to less experienced users, the Automatic Calibration Tool numgeo-ACT was developed. It is implemented in Python, allows the use of heuristic optimisation algorithms and is based on the free FE program numgeo. In this paper, the extension of ACT to the calibration of parameters for cyclic loading is presented for two advanced constitutive models, namely Hypoplasticity with Intergranular Strain and SANISAND. The automatic parameter calibration is performed based on the results of drained monotonic triaxial tests, oedometric compression tests and undrained cyclic triaxial tests on Karlsruhe Fine Sand. Details of the calibration procedure and a thorough comparison of simulations with experimental results is presented. It is shown that the automatic parameter calibration simplifies the determination of material parameters significantly and improves the accuracy of the constitutive model predictions.

Keywords: constitutive model, automatic parameter calibration, cyclic triaxial tests, heuristic optimisation algorithms

1 INTRODUCTION

Manually calibrating the parameters of constitutive soil models remains a difficult, time-consuming task which requires advanced knowledge of the model to be calibrated. Many of the parameters often have to be calibrated in an iterative process by back-calculating results from laboratory tests, since the parameters cannot always be determined by specific experiments or empirical equations. In this process, the parameters are continuously adjusted until a satisfactory agreement between simulation and experiment is achieved. Therefore, usually only one of the parameters is optimised while the other parameters are kept fixed. The Automatic Calibration Tool numgeo-ACT was developed to simplify and fasten the calibration process (Machaček et al., 2022). Furthermore, numgeo-ACT helps the user to evaluate when a best fit between simulation and experiment is achieved. In this paper, the performance of the ACT is demonstrated by the calibration of two sophisticated constitutive models namely Hypoplasticity (von Wollffersdorff, 1996) with Intergranular Strain extension (IGS) (Niemunis and Herle, 1997) and SANISAND-04 (Dafalias and Manzari, 2004).

The paper is structured as follows. Section 2 presents the experimental data, based on which the performance of the ACT is demonstrated. The general workflow of numgeo-ACT, the setup for the automatic parameter calibration and the used constitutive models are outlined in Section 3. Finally, the results of the automatic parameter calibration are presented in Section 4.

2 EXPERIMENTAL DATA

The performance of the ACT is investigated by calibrating the parameters for the so-called Karlsruhe Fine Sand based on the experimental data provided in Wichtmann and Triantafyllidis (2016a, 2016b). A total of two oedometric compression and five drained monotonic triaxial tests tests with varying initial relative density I_{D0} (initial void ratio e_0) and initial mean effective stress p_0 were considered (Table 1).

Table 1: Summary of experimental data of the drained monotonic triaxial tests (TMD) and oedometric compression tests (OE) used for the automatic parameter calibration.

Test	p_0 / kPa	e_0 / -	I_{D0} / -
TMD2	100	0.975	0.21
TMD6	50	0.88	0.46
TMD14	300	0.814	0.64
TMD15	400	0.799	0.68
TMD17	100	0.758	0.79
OE1	-	1.039	0.04
OE12	-	0.721	0.88

In addition, two undrained cyclic triaxial tests TCUI7 (e_0 = 0.800, medium dense/dense) and TCUI19 (e_0 = 0.761, dense) are considered (Table 2). Both cyclic tests start the cyclic loading from an isotropic state, i.e. TCUI7 starts the cyclic loading from p_0 = 200 kPa with a stress amplitude σ_1^{ampl} of 60 kPa while TCUI19 starts the cyclic loading from p_0 = 100 kPa with a stress amplitude of 40 kPa.

Table 2: Experimental data of the undrained cyclic triaxial tests used for the automatic parameter calibration.

Test	p_0 / kPa	σ_1^{ampl} / kPa	e_0 / -	I_{D0} / -
TCUI7	200	60	0.800	0.67
TCUI19	100	40	0.761	0.78

3 NUMGEO-ACT

numgeo-ACT[1] is a tool-set which allows the automatic parameter calibration of constitutive soil models based on different laboratory tests, such as oedometric compression tests, drained and undrained triaxial tests under monotonic and cyclic loading, respectively. The software is implemented in Python and uses the free FE program numgeo (www.numgeo.de, Machaček and Staubach, 2021) for the simulation of laboratory tests, by means of single element simulations (an axisymmetric element is used). The workflow of the ACT is given in Figure 1 and is explained in the following. At the beginning of the calibration process, element test simulations of the considered experiments are performed for calibration using an estimated set of parameters. Next, the results of the simulations are evaluated and compared to the results of the experiments. The discrepancy between the results of the simulations and the experiments are quantified by a discrete Fréchet distance (Eiter and Mannila, 1994) since this similarity measure has proven to be robust and leads to the best results (Machaček et al., 2022). To ensure that all variables have equivalent influence on the optimisation, a comparison is performed in scaled stress and strain planes according to Machaček et al. (2022). Based on the amount of considered tests, a weighted error ϵ is calculated and compared to a tolerance value TOL. An optimisation algorithm now iteratively improves the estimated material parameters within predefined limits with the goal of minimising the error ϵ. The calibration process is terminated when a predefined max. number of iterations is reached or a tolerance criterion is met.

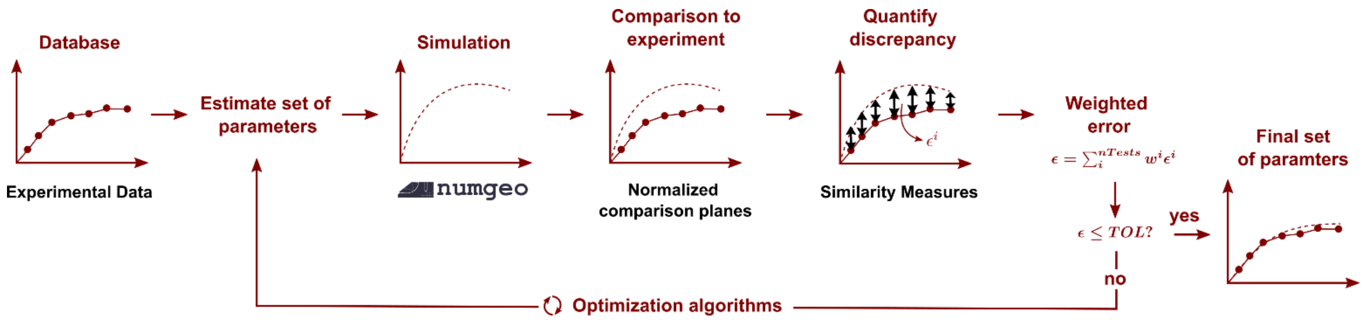

Figure 1: General workflow of numgeo-ACT

For the present work, a two stage approach for the automatic parameter calibration is followed: First, the parameters controlling the behaviour of the constitutive models under monotonic loading conditions are calibrated. Then, in a subsequent calibration run, the parameters for cyclic loading are determined. The optimisation algorithm applied in the present work corresponds to the JADE version (Zhang and Sanderson, 2009) of the Differential Evolution (DE, Storn and Price (1997)) algorithm, which improves the classical DE by an adaptive mutation scheme. The choice of the optimization algorithm is crucial for the the quality and reproducabiltiy of the automatic calibration. The good reproducability of DE was demonstrated in Machaček et al. (2022) by means of repeated calibration runs (up to 500) with the ACT.

3.1 Monotonic tests

For the calibration of the monotonic parameters the procedure presented in Machaček et al. (2022) is followed. The error from drained monotonic triaxial tests is calculated as $\epsilon^{CD} = 2/3\epsilon^{CD}_{\varepsilon q} + 1/3\epsilon^{CD}_{\varepsilon\varepsilon}$. Therein, $\epsilon^{CD}_{\varepsilon q}$ denotes the error in the normalised $\varepsilon_1 - q$ (ε_1: axial strain, q: deviatoric stress) plane and $\epsilon^{CD}_{\varepsilon\varepsilon}$ the error calculated in the normalised $\varepsilon_1 - \varepsilon_v$ (ε_v: volumetric strain) plane. The total error is defined as $\epsilon = \epsilon^{mon} = 1/2\epsilon^{CD} + 1/2\epsilon^{OC}$, with ϵ^{OC} being the error calculated in the normalised $\varepsilon_1 - \sigma_1$ (σ_1: axial stress) plane of oedometric compression tests. For the hypoplastic model, $\varphi_c = 33.1°$ was set corresponding to the critical friction angle of the material according to Wichtmann et al. (2019). The parameters $h_s, n, \alpha, \beta, e_{i0}, e_{c0}$ and e_{d0}

[1] https://j-machacek.github.io/numgeo-ACT/

were calibrated using the ACT. The parameters of the intergranular strain are switched off during the calibration of the monotonic tests due to the assumption of the intergranular strain to be fully mobilised in the loading direction. The parameters $M_e = \frac{6 \sin \varphi_c}{3+\sin \varphi_c}$ and $M_c = \frac{6 \sin \varphi_c}{3-\sin \varphi_c}$ of the SANISAND-04 model were determined based on the critical friction angle φ_c and held constant throughout the calibration. The parameters z_{max} and c_z were set to 20 and 2000, respectively. The remaining parameters $G_0, \nu, M_c, M_e, \lambda_c, e_0, \xi, m, h_0, c_h, n_b, A_0$ and n_d of the SANISAND-04 model were calibrated using the ACT.

3.2 Cyclic tests

For the calibration of the cyclic parameters, the focus was put on undrained cyclic triaxial tests. The error between the simulation and the experiment $\epsilon^{CUcyc} = 1/2\epsilon^{CUcyc}_{Np^w} + 1/2\epsilon^{CUcyc}_{N\varepsilon}$ is evaluated based on the following two planes: (a) $N - p_{w,acc}$ (N number of cycles, $p_{w,acc}$: accumulated pore water pressure) and (b) $N - \varepsilon_{1,mc}$ ($\varepsilon_{1,mc}$: mean values of axial strain calculated for each cycle separately). For the hypoplastic constitutive model, the calibration for cyclic loading focuses on the parameters of the intergranular strain extension m_R, R, β_R and χ. The parameter controlling the transverse stiffness increase was set to $m_T = 1/2\, m_R$. The parameters for the hypoplastic model correspond to those obtained for monotonic tests and remain unchanged. In the simulation of the cyclic tests, the intergranular strain tensor $\boldsymbol{h}$ was initialized as isotropically fully mobilised, i.e. $h_{ii} = -R/\sqrt{3}$.

A slightly modified strategy is followed for the SANISAND-04 model. Since an adjustment of the parameters c_z and z_{max} is not sufficient to achieve a satisfactory performance of the model under cyclic loading, an adjustment of some "monotonic" parameters is necessary. For this purpose, the parameters $G_0, m, h_0, \nu, n_b, n_d, A_0$ and c_h were also included in the calibration. This requires the monotonic tests to be included in the calibration in addition to the undrained cyclic triaxial tests. Obviously, this inevitably requires a trade-off of the models performance under monotonic and under cyclic loading. With the aim of finding an "overall best" parameter set, the errors for the cyclic and monotonic experiments were weighted equally: $\epsilon = 1/2\epsilon^{mon} + 1/2\epsilon^{cyc}$. Deviating from the “monotonic” calibration (see Section 3.1), tighter bounds were set for the monotonic parameters, which allow a change of the parameters by up to +100% and -50% of their initial value (obtianed from the ‘monotonic’ calibration). Due to the limitations of the SANISAND-04 model in capturing the elasto-plastic response under constant-η loading (Taiebat and Dafalias, 2007), the oedometric compression tests were excluded in the calibration of the models behaviour under cyclic loading. It should be noted, however, that the SANISAND-04 model can provide good results in reproducing oedometric compression tests if the calibration (and thus subsequent application) is limited to monotonic loading, see Figure 2 and Machaček et al. (2023). Details of the applied parameter bounds and the results of the calibrated parameters can be found in Section 4.

4 RESULTS OF THE AUTOMATIC PARAMETER CALIBRATION

In the following, the results of the automatically calibrated parameters are shown separately for the calibration based on monotonic tests and the subsequent calibration focusing on the models behaviour in cyclic tests. The values of the best fit parameters and corresponding parameter bounds are summarised in Table 3 and Table 4 for Hypoplasticity with IGS extension and for SANISAND-04, respectively.

4.1 Monotonic tests

The final results of the calibration based on drained monotonic triaxial tests and oedometric compression tests are given in Figure 2 for the hypoplastic model (red curves) and for SANISAND-04 (blue curves). The hypoplastic model tends to overshoot the deviatoric peak stresses of the drained monotonic triaxial tests, while the SANISAND-04 model captures the deviatoric peak stresses quite accurately. The initial stiffness and the dilatancy behaviour of the drained monotonic tests are better reproduced by the SANISAND-04 model than by the hypoplastic model.

Comparing the results of the back-analysis of the oedometric compression tests in Figure 2, it is evident that the stress strain behaviour simulated with the hypoplastic model gives a slightly better agreement with the experimental results than the SANISAND-04 simulations.

Overall, the comparison between the simulations and the experiments shows that the ACT is able to find parameter sets with which both models predict the experimental data of the drained monotonic triaxial tests and the oedometric compression tests sufficiently well.

4.2 Cyclic tests

The back-analysis of the undrained cyclic triaxial tests with the automatically calibrated best fit parameters are presented for the hypoplastic model with IGS extension and SANISAND-04 for the test TCUI7 in Figure 3 and for the test TCUI19 in Figure 4. The comparison between the simulations and the experiments is shown for the $p - q$ plane, the accumulated pore water pressure $N - p_{w,acc}$ plane and the mean values of axial strain $N - \varepsilon_{1,mc}$ plane. Inspecting the $p - q$ planes of both tests, it is evident that the predictions with the hypoplastic model with IGS extension are too contractive during the first cycle while the other cycles agree rather well with the experiments.

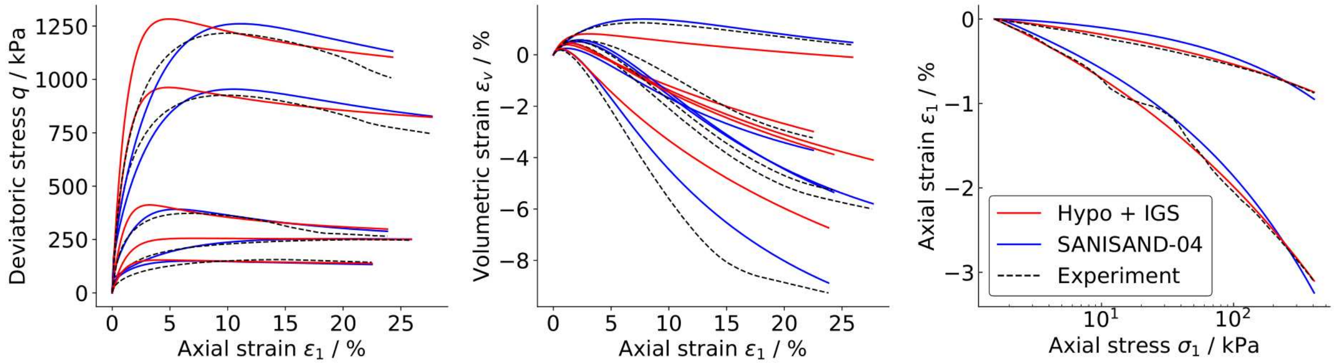

Figure 2: Results of the ACT for monotonic tests, Hypoplasticity with IGS extension (red curves) and SANISAND-04 (blue curves), left and centre: results from drained triaxial tests, right: results from oedometric compression tests

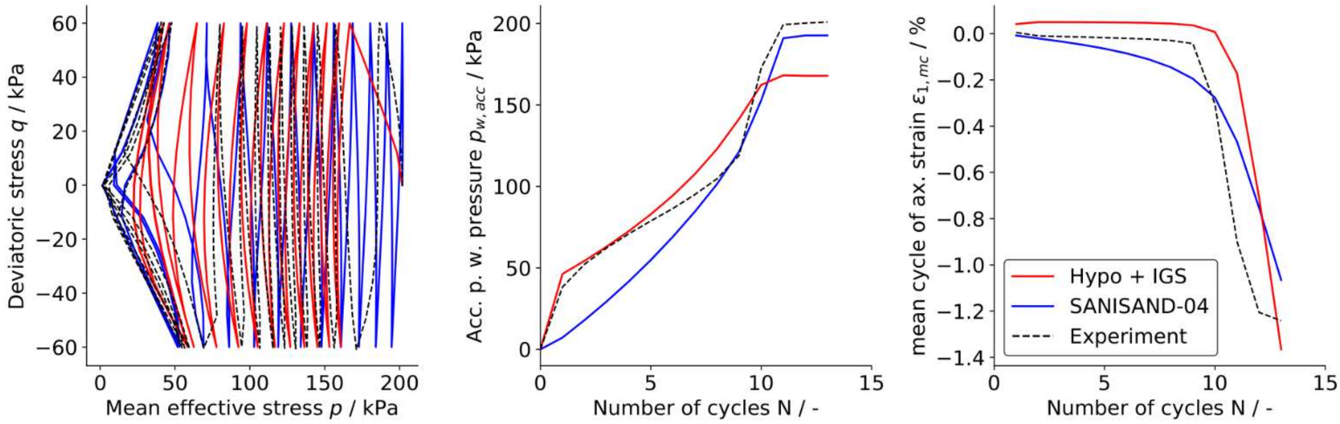

Figure 3: Results of the automatic parameter calibration for the undrained cyclic triaxial test ***TCUI7****, Hypoplasticity with IGS extension (red curves) and SANISAND-04 (blue curves), left: deviatoric stress q vs. mean effective stress p, centre: accumulated pore water pressure $p_{w,acc}$. vs. number of cycles N, right: mean values of axial strain $\varepsilon_{1,mc}$ vs. number of cycles N*

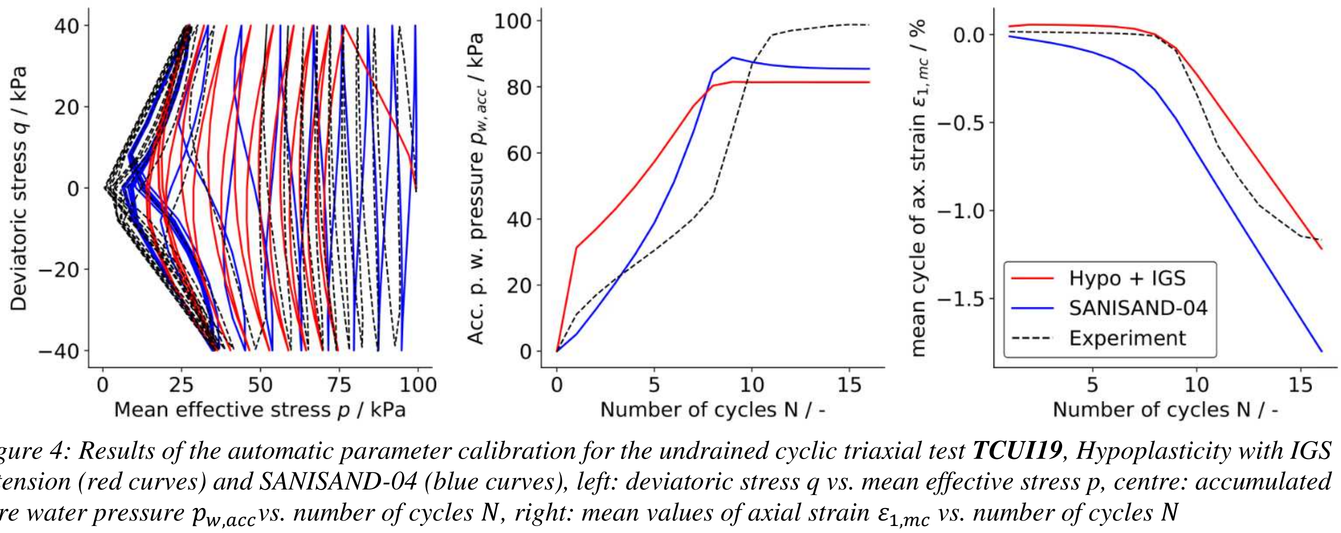

Figure 4: Results of the automatic parameter calibration for the undrained cyclic triaxial test ***TCUI19****, Hypoplasticity with IGS extension (red curves) and SANISAND-04 (blue curves), left: deviatoric stress q vs. mean effective stress p, centre: accumulated pore water pressure $p_{w,acc}$vs. number of cycles N, right: mean values of axial strain $\varepsilon_{1,mc}$ vs. number of cycles N*

Contrary, the SANISAND-04 model shows a clearly too stiff behaviour during the first cycle compared to the experiments. The hypoplastic model with IGS extension does not reach zero effective stress as observed in the experiments. Instead it shows a lens-shaped effective stress path at the end of the test, as also noticed in Wichtmann et al. (2019). The SANISAND-04 model, on the other hand, clearly shows a butterfly-like shape reaching almost zero effective stress in the simulation of the tests TCUI7 and TCUI19.

Generally, the trend of the accumulated pore water pressure and the mean values of axial strain are captured by both models. The predictions with the hypoplastic model with IGS extension are in good agreement with the test TCUI7, while the predicted porewater pressure accumulation rate of the test TCUI19 is clearly higher than measured.

The SANISAND-04 model tends to deliver a too rapid increase of the mean values of axial strain compared to the experiments in absolute values.

Summarised, numgeo-ACT is able to find suitable parameter sets for the prediction of the experimental data of the tests TCUI7 and TCUI19 for both the hypoplastic model with IGS extension and the SANISAND-04 model.

At this point, it is important to recall that in order to predict the cyclic tests sufficiently well, the drained triaxial tests were included in the calibration for cyclic loading of the SANISAND-04 model and some of the "monotonic" parameters were allowed to be optimised in predefined tighter bounds. This results in good cyclic test predictions as shown in Figure 3 and Figure 4. However, this approach significantly worsens the predictions of the monotonic tests as shown in Figure 5 (blue dashed curves).

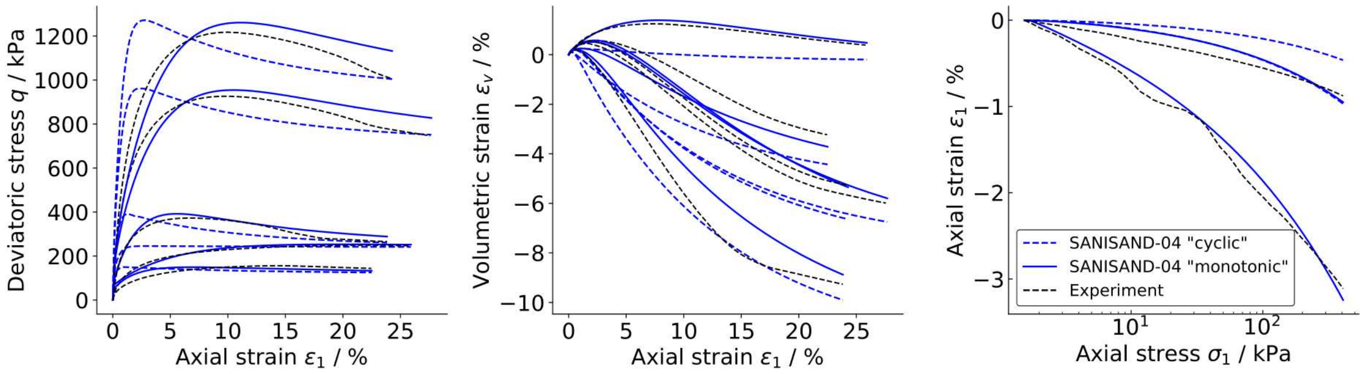

Figure 5: Results of the ACT for SANISAND-04, "cyclic": tests simulated with the parameter set which allows the best fit of the cyclic tests (blue dashed lines), "monotonic": tests simulated with the parameter set purely optimised on the monotonic tests (blue solid curves), left and centre: results from drained triaxial tests, right: results from oedometric compression tests

Through the adjustment of the parameters $G_0, m, h_0, \nu, n_b, n_d, A_0$ and c_h, in the calibration for cyclic loading, the SANISAND-04 model tends to overshoot the deviatoric peak stresses (similar to the hypoplastic model) and delivers a worse dilatancy behaviour of the drained monotonic tests. Furthermore, compared to the results of the oedometric compression tests, the predictions are clearly too stiff. For comparison, the results from Section 4.1, using the automatic parameter calibration purely based on monotonic tests are shown as blue solid curves in Figure 5.

The best fit parameters derived from the automatic parameter calibration are presented for the hypoplastic model with IGS extension in Table 3 and for SANISAND-04 in Table 4 together with the corresponding upper and lower parameter bounds which are applied in the automatic parameter calibration. The 3rd and 4th columns of Table 4 list the parameter bounds applied in the calibration of the SANISAND-04 model for monotonic loading (Section *3.1*), while the 5th and 6th columns list the tighter parameter bounds used in the calibration for cyclic loading (Section 3.2).

Again, it should be noted that the critical friction angle φ_c was not optimised. Hence the parameter φ_c of the hypoplastic model and the parameters M_c and M_e of the SANISAND-04 model are kept constant during the optimisation process.

Table 3. Results of the automatic parameter calibration, best fit parameters for the hypoplastic model (parameters $\varphi_c - \beta$) and for the IGS extension (parameters m_T – χ) with corresponding bounds

Parameter	value	upper bound	lower bound
φ_c / rad	0.577	-	-
h_s / GPa	27.87	45	1E-03
n / -	0.221	0.6	0.1
e_{d0} / -	0.609	0.744	0.609
e_{c0} / -	1.096	1.159	0.948
e_{i0} / e_{c0} / -	1.035	1.27	1.03
α / -	0.205	0.7	0.0
β / -	2.87	9.0	0.1
m_T / -	1.44	6.0	1.1
m_R / -	2.87	12.0	2.0
R / -	1.0E-04	2.5E-04	5.0E-05
β_R / -	0.09	1.4	0.05
χ / -	2.42	7	0.4

5 SUMMARY AND CONCLUSION

An automatic calibration tool numgeo-ACT was developed to calibrate the parameters of the constitutive models Hypoplasticity with Intergranular Strain (IGS) extension and SANISAND-04. The performance of the ACT was demonstrated based on the calibration of parameter sets for Karlsruhe fine sand considering five drained monotonic triaxial tests, two oedometric compression tests and two undrained cyclic triaxial tests with different initial densities and different initial mean effective stress.

Table 4. Results of the automatic parameter calibration for the SANISAND-04 model and corresponding bounds

		monotonic tests		cyclic tests	
Para-meter	value	upper bound	lower bound	upper-bound	lower bound
G_0 / kPa	212.3	200	50	253.9	63.5
υ / -	0.074	0.1	0.001	-	-
M_c / -	1.335	-	-	-	-
M_e / -	0.924	-	-	-	-
λ_c / -	0.005	0.25	0.0025	-	-
e_0 / -	0.985	1.8	0.6	-	-
ξ / -	1.031	1.0	0.2	-	-
m / -	0.040	0.05	0.005	-	-
h_0 / -	6.62	20	1.0	6.62	1.65
c_h / -	0.71	1.1	0.3	1.44	0.48
n_b / -	1.15	2.5	0.6	2.65	1.0
A_0 / -	0.66	1.4	0.2	1.05	0.35
n_d / -	3.20	4.0	0.5	3.2	1.0
z_{max} / -	36.02	-	-	60	1.0
c_z / -	399	-	-	10000	50.0

A two stage approach is chosen for the automatic parameter calibration of the hypoplastic model with IGS extension where the parameters for monotonic loading are calibrated first before the parameters for cyclic loading are addressed. For SANISAND-04, a slightly different approach is chosen, where monotonic tests are included in the calibration for cyclic loading and some of the "monotonic" parameters are allowed to be optimised in tighter bounds in order to achieve better results between the simulations and the cyclic tests. The results of the automatic parameter calibration show a good agreement between the simulations and the experiments for the hypoplastic model with IGS extension considering both monotonic and cyclic tests. For SANISAND-04 deviations between the simulations and the experiments of the oedometric compression tests were noticed. The other monotonic and cyclic tests are predicted sufficiently well by SANISAND-04. Overall, the automatic parameter calibration using numgeo-ACT has proven to be an efficient and reliable method to receive parameter sets for Hypoplasticity with IGS and SANISAND-04, which allows an optimised simulation of monotonic and cyclic soil behaviour.

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